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Multiobjective optimization for node adaptation in the analysis of composite plates using a meshless collocation method

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ABSTRACT

The bending of simply supported composite plates is analyzed using a direct collocation meshless numerical method. In order to optimize node distribution the Direct MultiSearch (DMS) for multi-objective optimization method is applied. In addition, the method optimizes the shape parameter in radial basis functions. The optimization algorithm was able to find good solutions for a large variety of nodes distribution.

1. Introduction

In this paper the Direct MultiSearch (DMS) for multiobjective optimization [1] is applied to the numerical modeling of composite plates in bending. A meshless numerical method is used (collocation with radial basis functions), with a third order shear deformation theory. In order to apply the numerical method a grid of N node centers is defined. The size of the matrix is directly related to the number of nodes (centers) in a grid. The objective in the optimization algorithm is to reduce the number of center nodes without compromising the accuracy of solutions. Since these can be conflicting objectives, a multiobjective optimization technique is used. The derivative-free solver DMS is applied to compute the Pareto front. The latter allows us to find a set of solutions also known as Pareto solutions. The set of Pareto solutions contains solutions with a wide range of errors and number of centers in a grid, allowing for the user to choose from that set of solutions a particular solution with a given number of nodes or a given error. Errors are defined by a residual $r = Pu - rhs$. In order to assess the accuracy of optimized solutions, solutions are interpolated in a regular node grid uniformly distributed over the domain and compared with analytical solutions.

In the collocation with radial basis function scheme it is assumed that any function ρ may be written as a combination of N continuously differentiable basis functions g:

$$\rho(\mathbf{x}) = \sum_{j=1}^{N} \chi_j g_j(\mathbf{x} - \mathbf{x}_j, \epsilon) \tag{1}$$

where g_j depends on a distance d between N grid nodes with coordinates $\mathbf{x}$, $\mathbf{x}_j$ is a node center and ϵ is a shape parameter. The shape parameter, sometimes referred as a 'fine tuner', is a non-zero input parameter defined by the user. The user defined shape parameter is a positive constant that may cause accuracy issues [2–5]. In this paper radial basis functions are used with Kansa's unsymmetrical collocation method [2,3]. This method produces dense, unsymmetrical, ill-conditioned matrices. High accuracy can be obtained if an adequate shape parameter is chosen.

Node distribution is also an important factor to influence the accuracy of solutions. Michelli demonstrated that multiquadric surface interpolation is always solvable, for distinct data sets [6]. Although any grid may be used, experience shows that different node distributions produce different results. Therefore, a given global error can be obtained with a different number of nodes and positions.

Some optimization techniques have been proposed to choose a good shape parameter. Rippa and Wang used a cross validation technique for shape parameter optimization in multiquadric interpolation [7,8]. The concept was extended by Roque and

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Ferreira to Kansa's method for solving systems of PDEs [9]. Using a cross validation technique it is possible to obtain good solutions for plate bending problems, even with a reduced number of grid nodes, for regular and irregular node distributions. In order to optimize node distribution for global collocation method with radial basis function, some proposed techniques use node adaptive grid strategies, usually using an error estimate to determine the node insertion/remotion strategy [10]. Sarra used node adaptive method for 1D time dependent partial differential equations [11]. Casanova et al. presented domain decomposition technique with a node adaptive algorithm to solve PDEs [12]. Hon et al. and Schaback and Wendland used an adaptive greedy algorithm to optimized node distribution when dealing with large radial basis functions systems [13,14]. An adaptive technique was also used by Hon for solving problems with boundary layer [15]. Shanazari and Hosami used an equi-distribution strategy to adapt node position for irregular regions [16] and Esmaeilbeigi and Hosseini introduce a dynamic algorithm to perform a local node adaptive strategy in nearly singular regions [17]. More recently, Uddin applied Rippa's algorithm to select a good shape parameter when solving time-dependent partial differential equations [18]. Iurlaro et al. developed an energy based approach for selecting a shape parameter and solved problems for the static deformation of rectangular, simply supported plates subjected to a bi-sinusoidal pressure [19].

In [20] the authors used the DMS algorithm to optimize grid node distribution and the shape parameter in the analysis of isotropic plates. In order to simplify the problem, node distribution and shape parameter were optimized independently. Although in [20] authors show how multiobjective optimization can be used to optimize plate in bending problems, the approach is limited to systems of differential equations with known analytical solution since the optimization procedure used the analytical solution in its formulation.

In the present approach, analytical solutions are not used in the objective function of the optimization problem, broadening the class of problems to be solved. The right-hand side (rhs) of the system is used to define a residual to be minimized. In addition the method optimizes the shape parameter for each optimized node distribution. As an example, a composite plate in bending under sinusoidal load is modeled. The problem of a plate in bending involves solving a system of partial differential equations with three distinct variables (w, ϕ_x and ϕ_y), corresponding to the plate vertical displacement, and the two rotations about x- and y-axes, respectively. The system is of the form $Pu = rhs$, where P represents differential operators, rhs contains external loads and u is the vector of solutions.

This paper is organized as follows. In Section 2 a review of the meshless numerical method is made. In Section 3 the third order shear deformation theory is presented. In Section 3 the DMS algorithm is developed. Numerical examples are presented in Section 5.

2. Global collocation for PDE

Consider a boundary problem with domain $\Omega \in \mathbb{R}^n$ and with an elliptic differential equation given by

$$\begin{cases} Hu(x) = s(x), & x \in \Omega \subset \mathbb{R}^n \\ Bu(x) = l(x), & x \in \partial\Omega \subset \mathbb{R}^n \end{cases} \tag{2}$$

where H and B are differential operators in domain Ω and in boundary $\partial\Omega$, respectively. Nodes $(\mathbf{x}_j, j = 1, \ldots, N_B)$ and $(\mathbf{x}_j, j = N_B + 1, \ldots, N)$ are distributed in the boundary and on the domain respectively. The solution $u(\mathbf{x})$ is approximated by $\tilde{u}$:

$$\tilde{u}(\mathbf{x}) = \sum_{j=1}^{N} \gamma_j g(\|\mathbf{x} - \mathbf{x}_j\|, \epsilon) \tag{3}$$

Inserting operators H and B in Eq. (3) the following equations are obtained:

$$\begin{cases} \tilde{u}_B(\mathbf{x}) \equiv \sum_{j=1}^{N} \gamma_j B g(\|\mathbf{x} - \mathbf{x}_j\|, \epsilon) = l(\mathbf{x}_i); & i = 1, \ldots, N_B \\ \tilde{u}_H(\mathbf{x}) \equiv \sum_{j=1}^{N} \gamma_j H g(\|\mathbf{x} - \mathbf{x}_j\|, \epsilon) = s(\mathbf{x}_i); & i = N_B + 1, \ldots, N \end{cases} \tag{4}$$

where $l(\mathbf{x}_i)$ and $s(\mathbf{x}_i)$ are the prescribed values on boundary nodes and domain nodes, respectively. Solving the previous system in the order of γ, it is possible to interpolate the solution by using Eq. (3).

In the present paper, the multiquadric radial basis function is considered:

$$g = \sqrt{r^2 + \epsilon^2} \tag{5}$$

where r is the Euclidean distance between distinct grid nodes and ϵ is a shape parameter.

For the present problem of a composite plate in bending, differential operators H and B are derived using the third-order shear deformation theory (TSDT) [21]. A brief overview of TSDT is given in Section 3.

3. Third-order shear deformation theory

Composite plates are one of the most significant applications of composite materials in the industry. Layers are stacked together to form thin or thick laminates. The problem of a plate in bending is illustrated in Fig. 1 where a load q is applied at the plate's top surface. The governing equations that rule the bending of the plate are developed from an assumed displacement filed. In the present case, a third-order shear deformation theory for displacements is assumed. The third-order theory of Reddy (TSDT) is based on the same assumptions than the classical and first-order plate theories, except that the assumption of straightness and normality of a transverse normal after deformation is relaxed by expanding the displacements (u, v, w) as cubic functions of the thickness coordinate, z. Fig. 2 illustrates the deformation of a transverse normal (note that for a symmetric plate, $u_0, v_0 = 0$ and therefore these may be removed from Eqs. (7)–(9)). The TSDT, for a symmetric plate, gives origin to a set of 3 partial differential equations with respective boundary equations.

The third order theory of Reddy (TSDT) for plates has been used many times in the study of composite plates. In this section, we

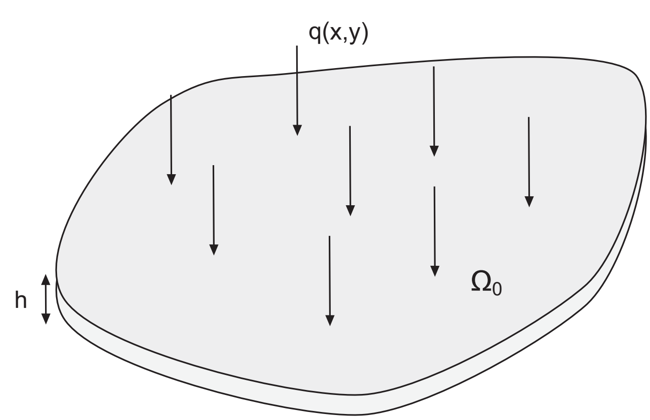

Fig. 1. Plate in bending problem.