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Free vibration analysis of stepped rectangular plates resting on non-homogeneous elastic foundations

M. Huang [a,*], T. Sakiyama [b], H. Matsuda [b], C. Morita [b]

[a] School of Nuclear Science and Engineering, North China Electric Power University, Beijing, China
[b] Department of Structural Engineering, Nagasaki University, Nagasaki 85, Japan

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ABSTRACT

A Half Boundary Method (HBM) is proposed for analyzing the free vibration problem of rectangular plates with stepped thickness resting on non-homogeneous elastic foundations. The unknown quantities of the method exist only on half of the boundary. The non-homogeneous elastic foundation discussed here consists of two-segment elastic Winkler foundation. The fundamental differential equations are established for the bending problem of the plate on elastic foundations. The Green function, which is obtained by transforming these differential equations into integral equations and using numerical integration, is used to establish the characteristic equation of the free vibration. The effects of the modulus of the foundation, the stepped thickness and aspect ratio on the frequency parameters are considered. By comparing the present numerical results with those previously published, the efficiency and accuracy of the present method are investigated.

1. Introduction

The free vibration problems of plates on the elastic foundations and plates with stepped thickness play an important role in engineering fields. Due to their flexible features of stiffness, strength, fundamental frequency parameter, etc., these plates can be found in many engineering applications, such as buildings and their foundations in civil engineering, printed circuit board design in electronic engineering and wings of aircrafts in aerospace engineering. For the plates with some specific boundary conditions, the analysis methods could be used to obtain the solutions. The Levy solution approach associated with the state space technique was used to obtain the exact solution of vibration of rectangular Mindlin plates resting on non-homogenous elastic foundations by Xiang [1]. The effect of the transverse shear deformation, rotary inertia and foundation stiffness parameter on frequency parameters was investigated. The same method was also used to obtain the exact buckling and free vibration solution of rectangular plates with two opposite edges simply supported and stepped variation in thickness by Xiang and Wang [2]. Lam et al. [3] obtained the exact solutions of the bending, buckling and vibration for rectangular plates on a two-parameter foundation. The Green's functions were employed and the numerical results were given for the plates with two opposite edges simply supported. Xiang et al. [4] used the Navier solution procedure to obtain the closed-form buckling and vibration solutions of simply supported symmetric cross-ply laminates on Pasternak foundations. The effects of the aspect ratios, in-plane load and the foundation parameters on the buckling factor and frequency parameter were presented. Akavci [5] used the same method to analyze free vibration of functionally graded thick rectangular plates on elastic foundation plates. The efficiency of an improved version of a hyperbolic shear deformation theory was investigated. Huu et al. [6] proposed a simple refined shear deformation theory for bending, buckling and vibration of thick plates resting on elastic foundation in which no shear correction factor was used. Liew et al. [7] presented a semi-analytical method for free vibration of plates with stepped thickness at the central part.

For the plates with general boundary conditions, the analysis methods were no longer effective and the numerical methods were used to analyze the problems of these plates. The Rayleigh–Ritz method was used to obtain the fundamental frequency of vibration of circular, regular polygonal and rectangular plates on non-homogeneous foundations by Laura and Guriérrez [8,9]. The same method was used to analyze the free and forced vibration of Reissner–Mindlin plates on elastic foundations by Shen et al. [10] and the free vibration of nonhomogeneous orthotropic rectangular plates with bilinear thickness variation resting on Winkler foundation by Kumar and Lal [11]. Horenberg and Kerstens [12] solved the same problem described in Ref. [9] by using the modal constraint

* Corresponding author. Tel.: +086 (0)10 61773163.
E-mail address: huangmei@ncepu.edu.cn (M. Huang).

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method. The fundamental frequency coefficients were presented for the plates with clamped and simply supported edges. By the same method, Omurtag and Kadioğlu [13] studied the free vibration of orthotropic plates resting on Pasternak foundation and presented some numerical results for plates with simply supported boundary conditions. Matsunaga [14] used the method of power series expansion of the displacement components to investigate the vibration and stability of thick plates on elastic foundation. Based on the higher-order theory of thick plate, the natural frequency and the buckling stress were given for a simply supported square plate on a two-parameter elastic foundation and subjected to in-plane stress. Huang and Thambiratnam [15] used the finite strip method to analyze the static and dynamic responses of plate resting on elastic supports and elastic foundation. A spring system was used to simulate these elastic supports and foundation. Jahromi et al. [16] applied the generalized differential quadrature method to analyze the vibration characteristics of moderately thick rectangular plate partially resting on Pasternak foundation. The effect of some parameters on the natural frequencies of the plates was investigated. Guo et al. [17] proposed a dynamic finite strip method for plates with stepped thickness. The results obtained by the proposed method and the finite element method were shown and discussed for simply supported plates. Harik et al. [18] also used the strip method to obtain the fundamental frequency of rectangular plates having single or multiple step variation in thickness. Liu and Huang [19] solved the free vibration problem of thick cantilever laminated plate with stepped variation in thickness. The three-dimensional finite element method and the transfer matrix method were employed. The effects of the fibre orientations, the layer arrangement and the geometrical parameters on the frequency parameters were discussed. Lal et al. [20] used the quintic splines interpolation technique to analyze the free vibration of tapered orthotropic rectangular plates on Winkler foundation. The numerical results were presented for plates with opposite edges simply supported. Ju et al. [21] analyzed the free vibration of rectangular and circular plates with stepped thickness resting on non-homogeneous elastic foundations by using the finite element method. Natural frequency parameters and mode shapes of these plates were presented. Based on Gâteaux differential, a mixed finite element formulation was derived and used to analyze the static and dynamic problems of thin plate on elastic foundation by Omurtag et al. [22] . The numerical results were obtained for clamped and simply supported plates with variable thickness on Winkler or Pasternak foundations.

In recent years, Mesh-free or Meshless methods have attracted much more attention and used extensively in engineering. Unlike mesh-based methods such as finite differential method, finite element method and boundary element method, mesh-free methods are based on a set of randomly distributed nodes that make them much easily solve the problems with discontinuities, moving boundaries, large deformations and so on. Variety of meshless or mesh-reduced methods have been introduced and applied for vibration analysis of plates [23–26]. But few have been found to solve the problem of free vibration of plates with foundations or with stepped thickness. Bahmyari and Khedmati [27] analyzed vibration of moderately nonhomogeneous moderately thick plates with point supports resting on Pasternak elastic foundation using element free Galerkin method. The effects of foundation parameters, various types of boundary conditions and different numbers of point support on the frequencies were investigated.

In this paper, a Half Boundary Method (HBM) is proposed for analyzing the free vibration of rectangular plates with stepped thickness resting on non-homogeneous elastic foundations. The method is a mesh-free method and based on the authors' early work [28,29]. Compared with the early work, HBM can determine the relationship between the internal nodes and pointed boundary nodes rapidly by using local regional integration. The characteristics of the method are as follows. (1) The unknown quantities exist only on half of the boundary. The region in which the unknown quantities exist is only half of that of boundary element method and other mesh-free boundary type method. Compared with district type methods such as finite element method and finite differential method, the region is reduced one dimension. That reduces the number of free degrees and the scale of the matrixes, therefore it can save CPU time and memory. (2) The method does not require the fundamental solutions which are needed in the boundary element method, the method of fundamental solutions, the boundary knot method, etc. The starting point is to establish fundamental differential equations expressed by mixed variables and no "shear-locking" phenomenon occurs. (3) The order of the differential equations is reduced to the first-order from the fourth-order by using mixed variables instead of deformation variables. That makes all the variables have the same accuracy. (4) The boundary conditions including twisting moment conditions can be satisfied exactly on the pointed boundary nodes. The efficiency and accuracy of the present method for the free vibration of stepped rectangular plates on Winkler foundation are investigated. The effects of the foundation modulus, the aspect ratio, the stepped variation in thickness on the frequency parameter are considered. Some numerical results are obtained for the plates with stepped thickness in one or two directions resting on non-homogeneous elastic foundations. The plates with stepped thickness in one direction which is not parallel to the plate edges are also studied.

2. Fundamental differential equations

Consider a rectangular plate of length a, width b, density ρ resting on a Winkler foundation of foundation modulus k_f as shown in Fig. 1. An xyz coordinate system is used in the present study with its $x-y$ plane contained in the middle plane of the rectangular plate, the z-axis perpendicular to the middle plane of the plate and the origin at one of the corners of the plate. The rectangular plate has stepped thickness in any direction.

In this paper, the elastic foundation is modeled as a spring system and the intensity of the reaction of the foundation is assumed to be proportional to the deflection w of the plate. Considering the equations of equilibrium, the strain–displacement relations, the stress–strain relations and the load–stress relations, the fundamental differential equations of the plate having a concentrated load $\overline{P}$ at a point (x_q, y_r) and resting on a Winkler foundation of the foundation modulus k_f are as follows:

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}+\overline{P}\delta(x-x_q)\delta(y-y_r)-k_f w=0,$$

$$\frac{\partial M_{xy}}{\partial x}+\frac{\partial M_y}{\partial y}-Q_y=0,$$

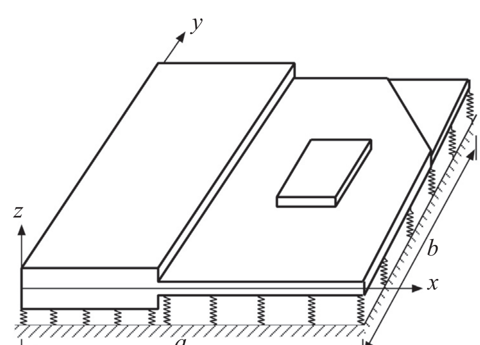

Fig. 1. A rectangular plate with stepped thickness resting on elastic foundations.