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Yield design of reinforced concrete slabs using a rotation-free meshfree method

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ABSTRACT

This paper presents a numerical kinematic procedure for yield design of reinforced concrete slabs governed by Nielsen's yield criterion that uses a rotation-free meshfree method and second-order cone programming. A moving least squares approximation technique is employed to approximate the transverse displacement field without using rotational degrees of freedom. A curvature smoothing stabilization technique is applied, ensuring that the size of the resulting optimization problem is reduced significantly. The resulting optimization was solved using a highly efficient primal-dual interior point algorithm. Various reinforced concrete slab problems with arbitrary geometries and different boundary conditions were solved to illustrate the efficacy of the proposed numerical procedure.

1. Introduction

The yield-line method is a long established and highly effective means of estimating the load bearing capacity of slabs and plates. This well-known method can predict very good upper-bound of the actual collapse multiplier for many practical engineering problems [1–3]. However, this hand-based analysis method encounters difficulties in problems of arbitrary geometry due to the need of postulating a collapse mechanism, which is generally unknown a priori. Consequently, numerical procedures based on finite element method and mathematical programming have been developed over the past few decades. Anderheggen and Knopfel [4] were amongst the first to perform yield design or limit analysis of slabs using finite elements and linear programming. Obviating the need to linearize the yield surface, non-linear optimization [5] and second-order cone programming techniques [6–11] can be applied to both kinematic and static formulations.

In the kinematic formulation for yield design of slabs, velocity fields may be discretized using discontinuous finite elements [12–14]. The element is assumed to be rigid with a constant angle of rotation along each edge of the element, and hence potential yield-lines will be placed at the boundaries of these rigid elements. Edge rotations can be approximated based on the out-of-plane nodal displacements, and there is no need of rotational degrees of freedom. The element is able to identify the most critical layout of yield-lines, however it was found to be very sensitive to the mesh layout. The so-called rotation-free elements (without rotational degree of freedom) were further developed by many others [15–17]. In these elements, curvature for each edge is interpolated based on the control element and its neighboring elements. Recently, Al-Sabah and Falter [18] have modified the *S*3 element presented in [17] by introducing yield-lines passing through the element. It has been shown that the modified *S*3 element can provide accurate collapse load multipliers with a relatively small number of degrees of freedom.

In the past decades, the so-called meshfree methods have been developed to provide a flexible alternative approach to the finite element method. The methods use sets of nodes distributed across the problem domain, and also along domain boundaries. Various rotation-free meshfree based models have been proposed for thin plate problems [19–21]. It has been demonstrated in [21] that constant curvature thin plate formulations using the radial point interpolation method in combination with a generalized gradient smoothing technique can result in stable and accurate solutions for static and free vibration analysis. Taking advantages of such a rotation-free meshfree formulation, in this paper a numerical procedure based on moving least squares approximation and curvature smoothing technique will be developed for yield design of slabs governed by Nielsen's yield criterion.

The layout of the paper is as follows. The next section will describe a kinematic formulation for yield design or limit analysis of reinforced concrete slabs governed by Nielsen's yield criterion.

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In Section 3, a moving least squares approximation technique is employed to discretized the transverse displacement field without using rotational degrees of freedom. A rotation-free meshfree based kinematic formulation is then described. Numerical examples are provided in Section 4 to illustrate the performance of the proposed procedure.

2. Kinematic limit analysis of reinforced concrete slabs

Consider a thin rigid-perfectly plastic plate bounded by a curve enclosing a plane area Ω with kinematic boundary Γ_u and static boundary Γ_t, and subjected to an out-of-plane load λq. The exact collapse multiplier λ_{exact} can be determined by solving any of the following optimization problems [6]

$$\lambda_{exact} = \max\{\lambda \,|\, \exists \mathbf{m} \in \mathcal{B} : a(\mathbf{m}, u) = \lambda F(u), \forall u \in Y\} \tag{1}$$

$$= \max_{\mathbf{m} \in \mathcal{B}} \min_{u \in \mathcal{C}} a(\mathbf{m}, u) \tag{2}$$

$$= \min_{u \in \mathcal{C}} \max_{\mathbf{m} \in \mathcal{B}} a(\mathbf{m}, u) \tag{3}$$

$$= \min_{u \in \mathcal{C}} D(u), \tag{4}$$

where $\mathcal{C} = \{u \in Y | F(u) = 1\}$, $D(u) = \max_{\mathbf{m} \in \mathcal{B}} a(\mathbf{m}, u)$ is the plastic dissipation rate, Y is a space of kinematically admissible velocity fields u, $\mathcal{B}$ is the yield condition and $F(u)$, $a(\mathbf{m}, u)$ are the external and internal virtual work respectively given by

$$F(u) = \int_\Omega qu \, d\Omega \tag{5}$$

$$a(\mathbf{m}, u) = \int_\Omega \mathbf{m}^T \boldsymbol{\kappa} \, d\Omega = -\int_\Omega \mathbf{m}^T \nabla^2 u \, d\Omega \tag{6}$$

In this study, the yield criterion proposed by Nielsen [22,3] and Wolfensberger [23], which is commonly known as Nielsen's yield criterion, is used for the analysis of reinforced concrete slabs. The criterion can be expressed as two rotated quadratic cones

$$\mathbf{b}_i + \mathbf{Q}_i \mathbf{m} \in \mathcal{K}_r^3, \quad i = 1, 2 \tag{7}$$

where

$$\mathbf{Q}_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix}; \quad \mathbf{b}_1 = \begin{bmatrix} m_{px}^+ \\ m_{py}^+ \\ 0 \end{bmatrix}; \quad \mathbf{Q}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix};$$
$$\mathbf{b}2 = \begin{bmatrix} m_{px}^- \\ m_{py}^- \\ 0 \end{bmatrix} \tag{8}$$

in which m_{px}^- and m_{py}^- are the negative yield moments in the x and y directions, respectively, and similarly m_{px}^+ and m_{py}^+ are the positive yield moments in the two directions.

The plastic dissipation can be determined by solving the following optimization problem

$$\begin{aligned} \max \quad & \kappa^T \mathbf{m} \\ \text{s.t} \quad & \mathbf{b}_i + \mathbf{Q}_i \mathbf{m} \in \mathcal{K}_r^3, \quad i = 1, 2 \end{aligned} \tag{9}$$

The dual optimization problem corresponding to (9) is

$$\begin{aligned} \min \quad & m_{px}^+ \kappa_x^+ + m_{py}^+ \kappa_y^+ + m_{px}^- \kappa_x^- + m_{py}^- \kappa_x^- \\ \text{s.t} \quad & \begin{cases} (\kappa_x^+, \kappa_y^+, \kappa_{xy}^+) \in K_r^{+(3)} \\ (\kappa_x^-, \kappa_y^-, \kappa_{xy}^-) \in K_r^{-(3)} \\ \kappa_x = \kappa_x^+ - \kappa_x^- \\ \kappa_y = \kappa_y^+ - \kappa_y^- \\ \kappa_{xy} = -\sqrt{2}(\kappa_{xy}^+ + \kappa_{xy}^-) \end{cases} \end{aligned} \tag{10}$$

The upper bound on the collapse load of reinforced concrete slabs can be then determined by the following mathematical programming [24]

$$\begin{aligned} \lambda^+ = \min \quad & D(u) = \int_\Omega a(u) \, d\Omega \\ & = \int_\Omega \left(m_{px}^+ \kappa_x^+ + m_{py}^+ \kappa_y^+ + m_{px}^- \kappa_x^- + m_{py}^- \kappa_x^- \right) d\Omega \\ \text{s.t} \quad & \begin{cases} (\kappa_x^+, \kappa_y^+, 2\kappa_{xy}^+) \in \mathcal{K}_r^{+(3)} \\ (\kappa_x^-, \kappa_y^-, 2\kappa_{xy}^-) \in \mathcal{K}_r^{-(3)} \\ \kappa_x = \kappa_x^+ - \kappa_x^- \\ \kappa_y = \kappa_y^+ - \kappa_y^- \\ \kappa_{xy} = -\sqrt{2}(\kappa_{xy}^+ + \kappa_{xy}^-) \\ \mathbf{u} = \mathbf{0} \text{ on } \Gamma_u \\ F(\mathbf{u}) = 1 \end{cases} \end{aligned} \tag{11}$$

where the last two constraints enforce boundary conditions and unitary external work. It should be noted that in the case when the velocity fields are assumed to be discontinuous across a line of discontinuity plastic dissipation produced by angular rotation discontinuities must be taken into account [9].

3. Rotation-free meshfree based kinematic formulation

3.1. Moving least squares approximation

The problem domain is discretized into a set of arbitrarily scattered points $\mathbf{x}_I$ $(I = 1, 2, \ldots, n)$. A moving least squares (MLS) approximation of the transverse displacement $u^h(\mathbf{x})$ using only nodal deflection u_I can be expressed as

$$u^h(\mathbf{x}) = \sum_{I=1}^{n} \Phi_I(\mathbf{x}) u_I \tag{12}$$

where the MLS shape functions $\Phi_I(\mathbf{x})$ are given as

$$\Phi_I(\mathbf{x}) = \mathbf{p}^T(\mathbf{x}) \mathbf{A}^{-1}(\mathbf{x}) \mathbf{B}_I(\mathbf{x}) \tag{13}$$

with

$$\mathbf{A}(\mathbf{x}) = \sum_{I=1}^{n} w_I(\mathbf{x}) \mathbf{p}(\mathbf{x}_I) \mathbf{p}^T(\mathbf{x}_I) \tag{14}$$

$$\mathbf{B}_I(\mathbf{x}) = w_I(\mathbf{x}) \mathbf{p}(\mathbf{x}_I) \tag{15}$$

where n is the number of nodes; $\mathbf{p}(\mathbf{x}) = [1, x, y, xy, x^2, y^2]^T$ is a quadratic basis function and $w_I(\mathbf{x})$ is an isotropic quartic spline weight function associated with node I.

Accordingly, rotations and curvatures can be obtained by directly differentiating the approximated function displacement $u^h(\mathbf{x})$

$$\theta_\alpha^h := u_{,\alpha}^h(\mathbf{x}) = \sum_{I=1}^{n} \Phi_{I,\alpha}(\mathbf{x}) u_I \tag{16}$$

$$\boldsymbol{\kappa}_{\alpha\beta}^h := u_{,\alpha\beta}^h(\mathbf{x}) = \sum_{I=1}^{n} \Phi_{I,\alpha\beta}(\mathbf{x}) u_I \tag{17}$$

Note that rotations calculated by Eq. (16) will be used when enforcing rotational boundary conditions only. It is also worth noting that it is often computationally expensive to calculate the second derivatives of shape functions in Eq. (17). In the following section, a technique that allows the required order of differentiation to be reduced by one will be presented. Consequently, there is no need to calculate the second derivatives of shape functions, but only first derivatives are needed.