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A local meshless method based on moving least squares and local radial basis functions

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ABSTRACT

In this paper, a local meshless method is presented, and this method is based on the linear combination of moving least squares and local radial basis functions in the same compact support domain, by changing the coefficient of the linear combination, the new method possesses the properties of moving least squares approximation and local radial basis functions, because of the local property of this method, it gives us the convenience of computing and it is suitable for practical problems. Numerical experiments are given to demonstrate the accuracy, effectiveness and feasibility of this method.

1. Introduction

In recent years, meshless methods [1] have developed rapidly, and they gradually become a class of powerful numerical methods. In these methods, mesh generation on the spatial domain is not necessary, only a set of scattered nodes are used to approximate the solution. So compared with the mesh dependent methods, it is easier to deal with the large deformation problem and the complicated domains problem. The meshless methods can be categorised into three groups according to the formulation procedures: meshless methods based on weak-forms [2], meshless methods based on collocation technique [3] and meshless methods based on the combination of weak-form and collocation techniques [1]. In these meshless methods, the construction of shape functions is the foremost problem, there are many constructing methods can be found in literature, such as the moving least squares (MLS) method, the partition of unity (PU) method, the radial basis functions (RBF) method, and so on. Among these methods, the MLS method and the RBF method are used frequently and successfully.

The MLS method is introduced by Lancaster and Salkauskas [4] for the surface construction, and the corresponding error analysis is discussed in [5–10]. In the MLS method, one can obtain a best approximation in a weighted least squares sense, and this method emphasizes the compacted support of weight function especially, so it has the local characteristics. However, it has some limitations, such as complex computation and lack of kronecker delta function property.

The RBF method [11] is a very efficient interpolating technique relating to the scattered data approximation, Wendland et al. give the error analysis in [12–15]. The RBF method has high precision, and it is very suitable for the scattered data model, moreover, it is very effective to solve high dimensional problems. However, there are some drawbacks, for example, the character of global supported. As the number of collocation points increases, the full matrix that obtained from discretization scheme is always ill-conditioned. In addition to this, it is very sensitive to select the free parameter c, this will lead to the inaccuracy of calculation.

In recent years, the RBF is used to develop the radial point interpolation (RPIM) method shape functions by Liu et al. [16,17], the RPIM method is a local method, and it is the method based on weak-forms. In 2003, the concept of local collocation method based on the RBF has been introduced by Lee et al. [18], in it, they present a meshless approximation strategy based on the local multiquadric and the local inverse multiquadric approximations, and they get the numerical solution of the poisson equation. The multiquadric and the inverse multiquadric are both the radial basis functions, so we denote this method by local radial basis functions (LRBF) method. The LRBF method can overcome the ill-conditioned problem and the sensitivity of the shape parameter in the RBF method. Moreover, in contrast to RBF, only scattered data in the neighboring points are used in LRBF, instead of using all the points, thus the order of the matrix which is obtained from discretization is reduced, so the matrix of shape function is sparse,

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which will improve the computational accuracy and be suitable for solving large-scale problems [19].

In the practical problems, interpolation and approximation are both very necessary, the approximation is in particular important if the noise exists in data. A collocation meshless method based on multiple basis functions is introduced by Mohamed et al. [20], in it, they use a linear interpolating function of both MLS and RBF for constructing shape function, but the method is global, the problems existed in the RBF are existence still. In this paper, we present a local meshless method based on the linear combination of MLS and LRBF in the same compact support, by changing the coefficient of the linear combination, this new method will change between interpolation and approximation. Because of the local characteristics, the method brings the convenience of computing, and it is very suitable for practical problems.

2. The shape function based on MLS and LRBF

In this section, we present the shape function based on the linear combination of MLS and LRBF, and give the error analysis and properties.

Let Ω be an open bounded domain in R^n, given data values $\{x_j, u_j\}$, $j=1,2,\ldots,N$, where x_j is the distinct scattered point in $\overline{\Omega}$, u_j is the data value of function u at the node x_j, N is the number of scattered nodes, and let u^h denote the approximate function of u in this work.

2.1. The outline of MLS, RBF and LRBF

In MLS method, the function u^h is produced in a weighted square sense, it can be defined as

$$u^h(x) = \sum_{i=1}^{m} p_i(x)a_i(x) = p^T(x)a(x), \quad \forall x \in \overline{\Omega}, \tag{1}$$

where m is the number of terms in the basis, $p_i(x)$ is the monomial basis function, $a_i(x)$ is the coefficient of the basis function, and

$$p^T(x) = [p_1(x), p_2(x), \ldots, p_m(x)], \tag{2}$$

$$a(x) = [a_1(x), a_2(x), \ldots, a_m(x)]. \tag{3}$$

The unknown coefficient $a(x)$ is determined by minimizing the functional J, which is defined as

$$J = \sum_{j=1}^{n} \omega(x-x_j)(u^h(x)-u_j)^2, \tag{4}$$

where n is the number of nodes in the support domain of the point x, $\omega(x-x_j)$ is the weight function, and x_j is the node in the influence domain of x.

Eq. (4) can be rewritten in the vector form

$$J = (Pa-u)^T W(Pa-u), \tag{5}$$

where

$$P = \begin{bmatrix} p_1(x_1) & p_2(x_1) & \cdots & p_m(x_1) \\ p_1(x_2) & p_2(x_2) & \cdots & p_m(x_2) \\ \vdots & \vdots & & \vdots \\ p_1(x_n) & p_2(x_n) & \cdots & p_m(x_n) \end{bmatrix}, \tag{6}$$

$$u = [u_1, u_2, \ldots, u_n]^T, \tag{7}$$

and

$$W = \begin{bmatrix} \omega(x-x_1) & 0 & \cdots & 0 \\ 0 & \omega(x-x_2) & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \omega(x-x_n) \end{bmatrix}. \tag{8}$$

According to the conditions (5)–(8), taking the derivative $a(x)$ to zero, we have

$$A(x)a(x) = B(x)u, \tag{9}$$

where

$$A(x) = P^T WP, \quad B(x) = P^T W. \tag{10}$$

Then, we get

$$a(x) = A^{-1}(x)B(x)u, \tag{11}$$

by substituting (11) into (1), we have

$$u^h(x) = p^T(x)a(x) = \Phi^T(x)u = \sum_{j=1}^{N} \phi_j(x)u_j, \tag{12}$$

where

$$\Phi^T(x) = p^T(x)A^{-1}(x)B(x) \tag{13}$$

and $\phi_j(x)$ is the shape function.

In RBF method, the interpolating function $u^h(x)$ can be written as

$$u(x) = \sum_{j=1}^{N} \lambda_j \phi(\|x-x_j\|_2) = \boldsymbol{\phi}^T \Lambda, \quad x \in \overline{\Omega}, \tag{14}$$

where λ_j is the unknown RBF coefficient, $\phi(\|x-x_j\|_2)$ is radial basis function, and

$$\boldsymbol{\phi} = [\phi(\|x-x_1\|_2), \phi(\|x-x_2\|_2), \ldots, \phi(\|x-x_N\|_2)]^T, \tag{15}$$

$$\Lambda = [\lambda_1, \lambda_2, \ldots, \lambda_N]^T. \tag{16}$$

In order to compute λ_j, assume that we want to interpolate the values $u(x_k)$, i.e.,

$$u(x_k) = \sum_{j=1}^{N} \lambda_j \phi(\|x_k-x_j\|_2) = u_k, \quad k=1,2,\ldots,N, \tag{17}$$

then leads to a linear system

$$\Phi\Lambda = U, \tag{18}$$

where

$$\Phi = \begin{bmatrix} \phi(\|x_1-x_1\|_2) & \cdots & \phi(\|x_1-x_k\|_2) & \cdots & \phi(\|x_1-x_N\|_2) \\ \vdots & & \vdots & & \vdots \\ \phi(\|x_k-x_1\|_2) & \cdots & \phi(\|x_k-x_k\|_2) & \cdots & \phi(\|x_k-x_N\|_2) \\ \vdots & & \vdots & & \vdots \\ \phi(\|x_N-x_1\|_2) & \cdots & \phi(\|x_N-x_k\|_2) & \cdots & \phi(\|x_N-x_N\|_2) \end{bmatrix}, \tag{19}$$

$$U = [u_1, u_2, \ldots, u_k, \ldots, u_N]^T. \tag{20}$$

So we can get the unknown coefficient Λ from the linear system (18),

$$\Lambda = \Phi^{-1}U, \tag{21}$$

by substituting (21) into (14), we get

$$u^h(x) = \boldsymbol{\phi}^T \Phi^{-1} U = \Psi U = \sum_{j=1}^{N} \varphi_j(x)u_j, \tag{22}$$

where

$$\Psi = \boldsymbol{\phi}^T \Phi^{-1}, \tag{23}$$

and $\varphi_j(x)$ is the shape function.

In LRBF method, the interpolating function $u^h(x)$ can also be written as

$$u(x) = \sum_{j=1}^{n} \lambda_j \phi(\|x-x_j\|_2) = \boldsymbol{\phi}^T \Lambda, \tag{24}$$