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Evidence-theory-based analysis for the prediction of exterior acoustic field with epistemic uncertainties

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ABSTRACT

Evidence theory has a strong ability to handle the epistemic uncertainty, based on which the uncertain parameters with limited information can be conveniently treated. In this paper, a numerical method is developed to predict the exterior acoustic field with epistemic uncertainties based on evidence theory. In order to alleviate the computational cost, the interval analysis technique is adopted to acquire the approximate frequency response amplitude bounds for each focal element, and the corresponding formulations of interval perturbation analysis for exterior acoustic field prediction are deduced. Inspired by the probability theory, the mean value, standard deviation and cumulative distribution are used to describe the distribution characteristics of evidence variables. Two numerical examples are given to illustrate the feasibility and effectiveness of the present method.

1. Introduction

With the increasing of people's awareness of the performance of NVH (noise, vibration and harshness), researches on the acoustic behavior of structural-acoustic systems have been undergone a rapid development in engineering recently [1,2]. The finite element method (FEM) and the boundary element method (BEM) are currently the most preferred tools for the low-frequency acoustic radiation problems [3,4]. Traditional acoustic radiation problems have been analyzed under the assumption that the physical properties, the applied loads and the boundary conditions are deterministic. However, due to the effects of manufacturing or construction tolerances, aggressive environment factors and unpredictable external excitations, uncertainties associated with the material properties, geometric dimensions, applied loads and other parameters are unavoidable. Quantifying, propagating and managing the concerned uncertainty are important, sometimes even imperative [5]. Uncertainty can be categorized into aleatory and epistemic types according to the source of uncertainty [6]. Aleatory uncertainty is the inherent variation associated with the physical system or the environment, which is always modeled as random variables or random processes using probability theory [7–9]. On the other hand, epistemic uncertainty derives from incomplete and imprecise knowledge or information in any phase or activity of the modeling process. The collection of more information or an increase of knowledge would help to reduce the level of uncertainty. Different kinds of theories have been developed to handle the epistemic uncertainty, including convex models [10–16], fuzzy sets [17–21], possibility theory [22,23] and evidence theory [24,25], etc. Convex models are developed for the cases where only the variation bounds of the uncertainty are available. In fuzzy sets theory, the membership function is used to characterize the input uncertainty. In possibility theory, evidences from different experts are no conflicting. In addition, some theories have been developed to handle the aleatory and epistemic uncertainty simultaneously, which include the p-box approach [26,27] and fuzzy probabilities [28,29]. In p-box approach, the left and right bounds on the cumulative probability distribution function are specified. Fuzzy probabilities deal with the situation when the outcomes of some random experiment are fuzzy sets.

Among the mentioned approaches above, evidence theory seems to be more general than other modeling techniques. Evidence theory has a much more flexible framework to quantify epistemic uncertainty from the perspective of its theoretical body. Under different cases, it can provide equivalent formulations to classical probability theory, possibility theory, p-box approach, fuzzy sets and convex models, respectively. Besides, it can deal with limited and even conflicting information from experts. Furthermore, the basic axioms of evidence theory allow us to combine aleatory and epistemic mixed uncertainties in a very natural way. Due to the above advantages, evidence theory has been widely used in artificial intelligence related fields, and it has been extended to conduct reliability analysis and structural static and dynamic response analysis recently. The strengths and weakness of evidence theory in reliability analysis were summarized by Oberkampf and Helton through a simple

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algebraic function [30]. An efficient method was proposed for evidence-theory-based reliability analysis using a multi-point approximation [31,32]. The evidence theory and Bayesian theory were compared for uncertainty quantification to test their effectiveness for decision-making problems [33]. The sampling-based sensitivity analysis and evidence theory were integrated for epistemic uncertainty in model inputs [34]. Bae et al. developed a sensitivity analysis technique for quantified uncertainty in evidence theory [35]. Mourelatos and Zhou proposed an efficient algorithm for evidence-based design optimization (EBDO) [36]. A unified uncertainty analysis approach was formulated to handle the mixture of aleatory and epistemic uncertainty [37]. Agarwal et al. proposed a multidisciplinary EBDO approach based upon the sequential approximate optimization strategy [38]. An EBDO approach through using a gradient projection technique was proposed by Alyanak et al. [39]. Three meta-modeling techniques for evidence-based reliability analysis were compared by Bai et al. [40]. Jiang et al. proposed a novel evidence-theory-based reliability analysis method [41], in which a concept of most probable focal element is proposed and based on it the computational cost of reliability analysis can be significantly reduced. A numerical method was proposed for static and dynamic analysis of structures with epistemic uncertainty through integrating FEM with evidence theory [42].

The analysis of acoustic radiation problem is always one of the key points in noise prediction, which is important for acquiring high level NVH performance in engineering design. In practical engineering, the parameters of the acoustic radiation system are usually involved with uncertainties. The evidence theory has a strong ability in uncertainty characterization because of its potential to handle both aleatory and epistemic uncertainties. From the overall perspective, some inspiring progresses have been made for structural uncertainty analysis and reliability-based design with evidence theory. However, the evidence-theory-based acoustic radiation problem has not been researched yet. Based on the characteristics of acoustic radiation system, an efficient evidence-theory-based analysis method is proposed in this paper for the response prediction of exterior acoustic fields with epistemic uncertainty.

The remainder of this paper is organized as follows. The fundamentals of evidence theory are introduced in Section 2. In Section 3, the equilibrium equation for exterior acoustic field prediction is established. In Section 4, an efficient analysis method to predict the response of exterior acoustic field with epistemic uncertainty is proposed. Two numerical examples are investigated in Section 5 and some conclusions are given in Section 6.

2. Evidence theory

2.1. Fundamentals of evidence theory

Evidence theory, also called as the Dempster–Shafer theory [20], was firstly proposed by Dempster and further developed by Shafer. The main concept of evidence theory is that our knowledge on a given problem can be inherently imprecise. Thus, an interval consisted of belief and plausibility is used to treat the uncertainty of the system response.

Evidence theory starts by defining a frame of discernment (FD), which consists of a set of mutually exclusive elementary propositions, and it can be viewed as a finite sample space in probability theory. For example, if a FD is given as $\Theta=\{x_1, x_2, x_3\}$, then x_1, x_2 and x_3 are mutually exclusive elementary propositions. 2^Θ is defined to denote the power set of Θ, which also indicate all the possible subset propositions of Θ and can be illustrated as follows

$$2^\Theta=\{\varnothing,\{x_1\},\{x_2\},\{x_3\},\{x_1,x_2\},\{x_1,x_3\},\{x_2,x_3\},\{x_1,x_2,x_3\}\} \tag{1}$$

In evidence theory, the probability is assigned not only to a single event but also to any subset of possible events, and this is one of the big differences between evidence theory and probability theory. As the most important concept in evidence theory, the basic probability assignment (BPA) expresses the degree of belief for a proposition. The BPA is assigned by making use of a mapping function m: $2^\Theta \to [0, 1]$, which should satisfy the following three axioms

$$\begin{aligned}&\text{Axioms 1}: m(A)\geq 0 \quad \text{for any } A\in 2^\Theta\\ &\text{Axioms 2}: m(\varnothing)=0\\ &\text{Axioms 3}: \sum_{A\in 2^\Theta} m(A)=1\end{aligned}$$

where $m(A)$ denotes the BPA corresponding to the event A, and the subset A satisfying $m(A)>0$ is called as focal element.

Sometimes the available evidence may come from independent sources or experts, and evidences of this style can be combined by using existing rules. For two BPAs $m_1(B)$ and $m_2(C)$, the combined evidence can be calculated by the Dempster' rule of combing [21].

$$m(A)=\frac{\sum_{B\cap C=A} m_1(B)m_2(C)}{1-K} \quad \text{for } A\neq 0 \tag{2}$$

where

$$K=\sum_{B\cap C=\varnothing} m_1(B)m_2(C) \tag{3}$$

In the above equation, K denotes the total conflict between two independent sources or experts. The Dempster' rule of combing filters out any conflicts or contradictions among the provided evidences. And it is usually appropriate for the evidence with relatively small amounts of conflict.

Due to the lack of knowledge or information, evidence theory cannot provide a precise value for a proposition A as in the probability theory. Therefore, it seems reasonable to use two measures, namely belief and plausibility, to quantify the lower and upper bounds of the precise probability. The two bounds of the interval [Bel(A), Pl(A)] are defined as following

$$\begin{aligned}\mathrm{Bel}(A)&=\sum_{C\subseteq A} m(C)\\ \mathrm{Pl}(A)&=\sum_{C\cap A\neq\varnothing} m(C)\end{aligned} \tag{4}$$

where Bel(A) is obtained by summing the BPAs of propositions which are totally included in A as a measure of belief, and Pl(A) is the summation of BPAs of propositions which are totally or partially include in A as a measure of plausibility.

2.2. Moments of a function with evidence variables

Considering a general function with q-dimensional independent variables

$$Y=g(\mathbf{A}), \quad A_i=1,2\ldots q \tag{5}$$

Similar to the joint PDF in probability theory, the joint frame of discernment S can be defined using the following Cartesian product

$$S=A_1\times A_2\times\cdots\times A_n=\{s_k=[a_1,a_2,\ldots,a_n], \quad a_j\in A_j, j=1,2,\ldots,q\} \tag{6}$$

where s_k denotes the focal element of the joint FD and a_j denotes the focal element of the jth evidence variable. Then a joint BPA can be defined as

$$m_s(s_k)=\begin{cases}\prod_{i=1}^{n} m(a_j)\\ 0 \quad \text{otherwise}\end{cases} \tag{7}$$

In the probability theory, distribution function can provide the entire statistical information of a random variable, and moments are employed to quantitatively measure the characteristics of a