Engineering Analysis with Boundary Elements 50 (2015) 435–446

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Enriched finite elements for branching cracks in deformable porous media

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ARTICLE INFO

Article history:
Received 11 February 2014
Received in revised form
30 June 2014
Accepted 17 September 2014
Available online 4 November 2014

Keywords:
Crack
Porous media
Finite element method

ABSTRACT

In this paper, we propose and verify a numerical approach to simulate fluid flow in deformable porous media without requiring the discretization to conform to the geometry of the sealed fractures (possibly intersecting). This approach is based on a fully coupled hydro-mechanical analysis and an extended finite element method (XFEM) to represent discrete fractures. Convergence tests indicate that the proposed scheme is both consistent and stable. The contributions of this paper include: (1) a new junction enrichment to describe intersecting fractures in deformable porous media; (2) the treatment of sealed fractures. We employ the resulting discretization scheme to perform numerical experiments, to illustrate that the inclination angles of the fractures and the penetration ratio of the sealed fractures are two key parameters governing the flow within the fractured porous medium.

1. Introduction

Natural fractures are common in sedimentary rocks and can be either open or cemented. In fractured oil/gas reservoirs, connected and open fractures are the dominant cause for fluid transport. Neuman [1] and Berkowitz [2] offer a comprehensive review of experimental, modeling and numerical methods for fluid transport in geological media with opened fractures. However, previous work rarely tackled the impact of sealed fractures, which is an important part of the porous matrix in most rocks [3]. Mineral deposits are believed to be the main cause of sealing (see [4] for a geological view on this phenomenon). With the further development of shale gas exploration, which has been taking place over several decades already, sealed fractures will most likely attract a recrudescent interest because they can be potentially reopened to become active flow paths after hydraulic fracturing [5]. In shale deposits, sealed fractures usually have high length/width aspect ratios and tend to be cemented by silicate and calcite [3,6]. Since the minerals filling the fractures have a much lower permeability than the surrounding rock, these fractures can be considered as impermeable walls, through which no fluid exchange is possible. This assumption is supported by experimental data, such as magnetic resonance imaging [7]. Concisely, the flow disturbances engendered by sealed fractures strongly contribute to the heterogeneity of porous media, and quantifying their effect on the flow is of considerable geological relevance and can provide valuable information to engineers when predicting flow rates.

A discrete fracture, or crack, is often modeled as a mechanical discontinuity, across which the displacements and pore pressure are discontinuous. When these cracks propagate, these discontinuities evolve within the domain. Several classes of methods are available to discretize mechanics problems involving such evolving discontinuities. The most traditional is the finite element method (FEM), in which elements and nodes have to be placed along the crack surface and "special" elements are used to represent the crack tip singularity when the latter arises, e.g. when the bulk material is assumed linear elastic [8]. Other methods have emerged, based on the partition of unity method [9,10], namely, the enriched/extended finite element methods (XFEM) [11,12], generalized finite element methods [13], the smoothed extended finite element method [14–17], the intrinsic

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http://dx.doi.org/10.1016/j.enganabound.2014.09.010
0955-7997/

Nomenclature

$\mathbf{a}_J$	nodal enriched degrees of freedom vector associated with the Heaviside function
$\mathbf{b}_{\alpha K}$	nodal enriched degrees of freedom vector associated with Branch functions
$\mathbf{B}$	strain operator
$B_l(\mathbf{x})$	near-tip function
$\mathbf{c}_M$	nodal enriched degrees of freedom vector associated with the Junction function
$\mathbf{D_e}$	elastic matrix
E	Young modulus
$\mathbf{f}_u$	force vector of displacement
$\mathbf{f}_p$	force vector of pressure
$\mathbf{H}$	permeability matrix
$H(\mathbf{x})$	Heaviside function
$\mathbf{I}$	unit tensor
I	serial number for the standard nodes
J	serial number for the Heaviside enriched nodes
$J(\mathbf{x})$	Junction function
$\mathbf{k}$, k	matrix permeability
$\mathbf{L}$	the differential operator
$\mathbf{L_c}$	the differential operator
$\mathbf{m}$	unit vector
$\mathbf{n}$	normal vector
$\mathbf{N}_u(\mathbf{x})$	standard shape function for displacement approximation
$\mathbf{N}_p(\mathbf{x})$	standard shape function for pressure approximation
$\widehat{N}_u(\mathbf{x})$	enriched shape function for displacement approximation
$\widehat{N}_p(\mathbf{x})$	enriched shape function for pressure approximation
n_{cr}	number of fractures
$\aleph$	the set of all nodes in the mesh
$\aleph_{cr}$	the set of nodes whose supports are cut by a fracture
$\aleph_{jun}$	the set of the nodes whose supports contain a junction
$\aleph_{tip}$	the set of the nodes whose supports contain a tip
$\overline{p}$	the prescribed pressure on the boundary
$\overline{p}_m$	Pore pressure
$\mathbf{P}$	pore pressure vector
$\overline{q}$	the volumetric flux on the boundary
R_p	fracture penetration ratio
t	time
$\bar{\mathbf{t}}$	the prescribed force on the boundary
Δt_n	time step for computation
$\mathbf{u}$	displacement vector of single point
$\mathbf{U}$	displacement vector
$\mathbf{w}$, $\mathbf{w}^*$	the weighting functions
$\mathbf{x}$	the coordinate of domain
α	declined angle of single crack
α_m	Biot coefficient
Γ_p	the prescribed pressure boundary
Γ_t	the prescribed force boundary
Γ_u	the prescribed displacement boundary
Γ_q	the prescribed flux boundary
$\boldsymbol{\varepsilon}$	total strain
$\boldsymbol{\varepsilon}_\mathbf{e}$	effective strain
$\boldsymbol{\varepsilon}_\mathbf{v}$	volumetric strain
θ	cracks intersection angle
μ	fluid viscosity
υ	Poisson's ratio
ρ	fluid density
$\boldsymbol{\sigma}$	matrix stress
φ	matrix porosity
Ω	computational domain

XFEM [18] and enriched meshfree methods [19]. A review of the treatment of cracks by partition of unity methods is provided in [20]. Partition of unity enrichment enables arbitrary functions to be added to the approximation space, e.g. functions which are discontinuous through the crack faces, or singular functions to represent the near-tip behavior. The major advantage of such partition of unity enrichment schemes is that discontinuities can be represented within finite elements, or meshfree domains of influence, thereby avoiding re-meshing. The possible addition of singularities also enables the use of coarser elements in the vicinity of the crack tip.

FEM with interface elements (IEs) are widely used in geotechnical engineering, e.g. specially designed elements such as Goodman's joint elements [21] may be used to represent a fracture. The IEs are used widely to tackle fracture mechanics problems but also hydro-mechanical problem with preexisting fractures [22,23]. Recently, Nick et al. [24] proposed a hybrid finite-element finite-volume method to simulate solute transport in discrete fractures and a surrounding porous matrix. They also compared three discretization schemes suitable for discrete fracture modeling [24].

Another fracture modeling method, lattice models, typically used to simulate fractures in cement and other random materials [25], was recently developed to describe the fluid flow in concrete with meso-scale cracks [26]. The advantage of this method is its capacity of heterogeneous simulation.

Herein, we propose to follow the route of partition of unity enrichment, where discontinuities, such as natural and artificial fractures, faults, and interfaces between rock layers, can be represented without requiring the mesh to conform to the, potentially complex, geometry of these evolving discontinuities. Recently, the enriched approximations were used to model flow within a fractured porous medium. Réthoré et al. [27,28] first proposed a two-scale numerical model to simulate fluid flow in fractured, deforming porous media. In this work, the discrete fractures open and grow independently. Watanabe et al. [29] developed a numerical scheme combining XFEM with lower-dimensional interface elements to solve coupled hydro-mechanical problems in discretely fractured porous media. However, in this approach, the lower-dimensional elements have to be located on the edge of solid continuum elements. Lamb et al. developed a numerical scheme integrating a fracture mapping technique with XFEM to capture the fluid flow and rock deformation in the fractured porous media [30]. Huang et al. [31] proposed a new enrichment scheme to model weak discontinuities across a fracture. In these models, the pore pressure of the rock matrix is treated as a weakly discontinuous field, which means the local pressure across fractures is continuous but the pressure gradient is discontinuous. Refer to [32,33] for details on strong and weak discontinuities. However, for sealed fractures, the pressure across cemented cracks is itself discontinuous and hence, strong discontinuities must be incorporated in the approximation.

In this paper, we propose a numerical framework to solve the coupled hydro-mechanical problem associated with flow in cracked porous media, as well as represent the effect of the geologically important sealed fractures. We develop a partition of unity enrichment technique where the pressure field is allowed to be discontinuous across sealed fractures. We also propose a simple approach to handle branching cracks as well as multiple junctions where several cracks meet at a single point. Numerical examples are used to verify the appropriateness of the discretization space to solve the coupled problem. The paper also presents a parametric study on the effects of sealed fractures on fluid flow in porous media. The remainder of this manuscript is outlined as follows. First, we define the problem and summarize the governing equations in