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A cell-based smoothed radial point interpolation method (CS-RPIM) for three-dimensional solids

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ABSTRACT

A cell-based smoothed radial point interpolation method (CS-RPIM) is formulated for three-dimensional elasticity problems. In present method, the problem domain is firstly discretized by tetrahedron background cells, and each tetrahedron cell is then further divided into several smoothing cells. The displacement field function is approximated using RPIM shape functions which have Kronecker delta function property. Supporting node selection for shape function construction uses the efficient T2L-scheme associated with the background cells. The smoothed Galerkin weak form is employed to create discretized system equations, and then the gradient smoothing operation is adopted to construct smoothed strain fields in every smoothing cell. Numerical examples are used to examine the present method in terms of accuracy, convergence, and efficiency. Compared with the finite element method using linear interpolation and node-based smoothed finite element method, the CS-RPIM solutions can achieve better efficiency, higher accuracy, and greater stability in static and free vibration analysis in presented examples.

1. Introduction

Mesh-free methods have been proposed and achieve remarkable progress in recent years, including smooth particle hydrodynamics [1,2], diffuse element method (DEM) [3], element free Galerkin (EFG) method [4], reproducing kernel particle method (RKPM) [5], finite point method (FPM) [6], meshless local Petrov–Galerkin (MLPG) method [7] et al.

The radial point interpolation method (RPIM) is a mesh-free method based on Galerkin weak form. In this method, the shape functions are constructed using simple interpolation through a set of nodes located in a local support domain. For different basis functions adopted, two types of PIM have been developed, i.e. polynomial PIM using polynomial basis functions [8–10] and radial PIM (RPIM) using radial basis functions [11–13]. The RPIM shape functions created using local irregular nodes are preferred in many ways, because (1) they have the Kronecker delta function property, which allows the straightforward imposition of essential boundary conditions and (2) very irregularity distributed nodes can be used.

The RPIM has been studied and developed by various means [14–16] and have been applied in various problems [17,18]. Recently, a scheme of cell-based smoothed solution has been proposed by Liu [19] by incorporating meshfree techniques with the standard FEM, and then it has been referenced and applied into meshfree areas [20,21]. In [21], CS-RPIM has been formulated for 2-D elastic problems and shows good performance.

In this paper, the CS-RPIM is extended for 3-D problems. In this method, a background cell of four-node tetrahedrons is employed and shape functions are constructed using linear polynomials, as tetrahedrons can be created by standard routines automatically for 3-D solids. Each background cell is then further divided into several smoothing domains, in which the gradient strain smoothing technique is adopted to construct smoothed strain fields. Therefore, the integration to compose stiffness matrix contains only shape functions instead of shape function gradients. To examine the performance of the proposed method, a series of benchmark examples is presented, and excellent results are obtained demonstrating the efficiency and accuracy of the present CS-RPIM schemes.

2. Theoretical basis

2.1. Radial point interpolation method

The radial point interpolation method is a series representation for meshfree function approximation using a set of arbitrarily

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distributed nodes inside a local support of an interested point. In the present formulation, the problem domain is firstly discretized with some scattered nodes, and then the background local supports are constructed using the tetrahedral mesh. Consider a function $u(\mathbf{x})$ defined in a 3D problem domain Ω. The function can be approximated in a local support domain of the point of interest $\mathbf{x}$ with a set of arbitrarily distributed nodes using radial basis function $R_i(\mathbf{x})$ augmented with polynomial basis function $P_j(\mathbf{x})$.

$$u(\mathbf{x}) = \sum_{i=1}^{n} R_i(\mathbf{x})a_i + \sum_{j=1}^{m} P_j(\mathbf{x})b_j = \mathbf{R}^T(\mathbf{x})\mathbf{a} + \mathbf{P}^T(\mathbf{x})\mathbf{b} \tag{1}$$

where a_i is the unknown coefficient for function $R_i(\mathbf{x})$, b_j the coefficient for $P_j(\mathbf{x})$, n the number of RBFs and is also identical to the number of nodes in the local support domain of the point of interest $\mathbf{x}$, and m determined according to the polynomial basis selected. When $m=0$, pure RBFs are used. Otherwise, the RBF is augmented with m terms of polynomial basis functions. In the present work, the multiquadrics RBF (MQ-RBF) [12,22] is used, which has the following form:

$$R_i(\mathbf{x}) = [(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2 + (\alpha_c d_c)^2]^q \tag{2}$$

where q and α_c are two shape parameters, which are real and arbitrary and have been examined in detail by Liu [9,12], d_c is the equivalent length of the background cell.

In order to determine the constants a_i and b_j, Eq. (1) is enforced to be satisfied at these n nodes in the local support domain, which leads to a set of n equations. The matrix form of these equations can be expressed as

$$\mathbf{U}_s = \mathbf{R}_q\mathbf{a} + \mathbf{P}_m\mathbf{b} \tag{3}$$

where the vector of function values $\mathbf{U}_s$ is

$$\mathbf{U}_s = \{ u_1 \quad u_2 \quad \cdots \quad u_n \}^T \tag{4}$$

$\mathbf{R}_q$ is the moment matrix of RBFs which can be expressed as

$$\mathbf{R}_q = \begin{bmatrix} R_1(\mathbf{x}_1) & R_2(\mathbf{x}_1) & \cdots & R_n(\mathbf{x}_1) \\ R_1(\mathbf{x}_2) & R_2(\mathbf{x}_2) & \cdots & R_n(\mathbf{x}_2) \\ \vdots & \vdots & \vdots & \vdots \\ R_1(\mathbf{x}_n) & R_2(\mathbf{x}_n) & \cdots & R_n(\mathbf{x}_n) \end{bmatrix}_{n\times n} \tag{5}$$

and the polynomial moment matrix $\mathbf{P}_m$ is

$$\mathbf{P}_m = \begin{bmatrix} p_1(\mathbf{x}_1) & p_2(\mathbf{x}_1) & \cdots & p_m(\mathbf{x}_1) \\ p_1(\mathbf{x}_2) & p_2(\mathbf{x}_2) & \cdots & p_m(\mathbf{x}_2) \\ \vdots & \vdots & \vdots & \vdots \\ p_1(\mathbf{x}_n) & p_2(\mathbf{x}_n) & \cdots & p_m(\mathbf{x}_n) \end{bmatrix}_{n\times m} \tag{6}$$

The vectors of unknown coefficients have the following form

$$\mathbf{a}^T = \{ a_1 \quad a_2 \quad \cdots \quad a_n \} \tag{7}$$

$$\mathbf{b}^T = \{ b_1 \quad b_2 \quad \cdots \quad b_n \} \tag{8}$$

Since there are $(n+m)$ unknowns in Eq. (3), so m additional equations need to be added using the following constraint conditions [23]

$$\sum_{i=1}^{n} p_j(\mathbf{x}_i)\mathbf{a}_i = \mathbf{P}_m^T\mathbf{a} = 0, \quad j = 1, 2, \ldots, m \tag{9}$$

Combining Eqs. (3) and (9) yields the following set of equations in the matrix form

$$\tilde{\mathbf{U}}_s = \begin{Bmatrix} \mathbf{U}_s \\ 0 \end{Bmatrix} = \begin{bmatrix} \mathbf{R}_q & \mathbf{P}_m \\ \mathbf{P}_m^T & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{a} \\ \mathbf{b} \end{Bmatrix} = \mathbf{G} \begin{Bmatrix} \mathbf{a} \\ \mathbf{b} \end{Bmatrix} \tag{10}$$

Solving Eq. (10) yields

$$\begin{Bmatrix} \mathbf{a} \\ \mathbf{b} \end{Bmatrix} = \mathbf{G}^{-1}\tilde{\mathbf{U}}_s \tag{11}$$

So the field variable $u(\mathbf{x})$ can be expressed as

$$u(\mathbf{x}) = \begin{bmatrix} \mathbf{R}_q^T(\mathbf{x}) & \mathbf{P}_m^T(\mathbf{x}) \end{bmatrix} \mathbf{G}^{-1}\tilde{\mathbf{U}}_s = \tilde{\mathbf{\Phi}}\tilde{\mathbf{U}}_s \tag{12}$$

where

$$\tilde{\mathbf{\Phi}}(\mathbf{x}) = \begin{bmatrix} \mathbf{R}_q^T(\mathbf{x}) & \mathbf{P}_m^T(\mathbf{x}) \end{bmatrix} \mathbf{G}^{-1} = \begin{Bmatrix} \varphi_1(\mathbf{x}) & \cdots & \varphi_n(\mathbf{x}) & \varphi_{n+1}(\mathbf{x}) & \cdots & \varphi_{n+m}(\mathbf{x}) \end{Bmatrix} \tag{13}$$

Finally, the RPIM shape functions $\mathbf{\Phi}(\mathbf{x})$ are obtained as

$$\mathbf{\Phi}^T(\mathbf{x}) = \begin{Bmatrix} \varphi_1(\mathbf{x}) & \varphi_2(\mathbf{x}) & \cdots & \varphi_n(\mathbf{x}) \end{Bmatrix} \tag{14}$$

An approximation in Eq. (12) can be rewritten as

$$u(\mathbf{x}) = \mathbf{\Phi}^T(\mathbf{x})\mathbf{U}_s = \sum_{k=1}^{n} \varphi_k u_k \tag{15}$$

The present shape function possess the reproducing properties due to the addition of polynomial basis, also satisfy the delta function properties, and always exist because of the adoption of RBFs.

2.2. Global weak form for three dimensional solids

Consider a 3-D solid mechanics problem defined in domain Ω bounded by Γ, which can be expressed by the following equation [24]

$$\mathbf{L}^T\boldsymbol{\sigma} + \mathbf{b} = 0 \quad \text{in} \quad \Omega \tag{16}$$

$$\mathbf{u} = \bar{\mathbf{u}} \quad \text{on} \quad \Gamma_u \tag{17}$$

$$\boldsymbol{\sigma}\cdot\mathbf{n} = \bar{\mathbf{t}} \quad \text{on} \quad \Gamma_t \tag{18}$$

where $\mathbf{L}$ is a differential operator in the following form

$$\mathbf{L}^{\mathrm{T}} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial z} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \tag{19}$$

$\boldsymbol{\sigma}^{\mathrm{T}} = \begin{Bmatrix} \sigma_{xx} & \sigma_{yy} & \sigma_{zz} & \sigma_{xy} & \sigma_{yz} & \sigma_{zx} \end{Bmatrix}$ is the stress vector, $\mathbf{b}^{\mathrm{T}} = \begin{Bmatrix} b_x & b_y & b_z \end{Bmatrix}$ is the body force vector; $\mathbf{u}$ is the displacement vector, and $\bar{\mathbf{u}}$ is the prescribed displacement on the essential boundaries; $\bar{\mathbf{t}}$ is the prescribed traction on the natural boundaries, and $\mathbf{n}$ is the vector of unit outward normal.

The standard Galerkin weak form can be expressed as

$$\int_{\Omega} (\mathbf{L}\delta\mathbf{u})^T(\mathbf{D}\mathbf{L}\mathbf{u})d\Omega - \int_{\Omega} \delta\mathbf{u}^T\mathbf{b}d\Omega - \int_{\Gamma_t} \delta\mathbf{u}^T\bar{\mathbf{t}}d\Gamma = 0 \tag{20}$$

where $\mathbf{D}$ is the matrix of material constants.

Substituting Eq. (1) into Eq. (20), the discretized system equation can be expressed in the following matrix form

$$\mathbf{K}\mathbf{d} = \mathbf{f} \tag{21}$$

where

$$\mathbf{K}_{ij} = \int_{\Omega} \mathbf{B}_i^{\mathrm{T}}\mathbf{D}\mathbf{B}_j d\Omega, \quad (i,j = 1, \ldots, N_{node}) \tag{22}$$

$$\mathbf{d} = \begin{Bmatrix} u_1 & v_1 & w_1 & \cdots & u_{N_{node}} & v_{N_{node}} & w_{N_{node}} \end{Bmatrix}^{\mathrm{T}} \tag{23}$$

$$\mathbf{f}_i = \int_{\Omega} \varphi_i\mathbf{b}d\Omega + \int_{\Gamma_t} \varphi_i\bar{\mathbf{t}}d\Gamma, \quad (i = 1, \ldots, N_{node}) \tag{24}$$

where N_{node} is the number of nodes in analysis, ϕ_i denotes the shape function and $\mathbf{B}_i$ can be given as