Engineering Analysis with Boundary Elements 41 (2014) 10–17

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Iterative simulation of 3D heat diffusion in a medium with multiple cracks

C. Serra [a,*], A. Tadeu [b], N. Simões [b]

[a] ITeCons—Instituto de Investigação e Desenvolvimento Tecnológico em Ciências da Construção, Rua Pedro Hispano s/n., 3030-289 Coimbra, Portugal
[b] Department of Civil Engineering, University of Coimbra, Pólo II, Rua Luís Reis Santos, 3030-788 Coimbra, Portugal

A R T I C L E I N F O

Article history:
Received 19 August 2013
Accepted 28 December 2013
Available online 25 January 2014

Keywords:
Point heat source
3D heat diffusion
Numerical modelling
Normal-derivative equation
Iterative TBEM
Multi-inclusions

A B S T R A C T

This paper presents an iterative three-dimensional (3D) normal-derivative equation model (TBEM) to simulate 3D heat diffusion generated by a point heat source in the presence of 3D cracks embedded in an unbounded spatially uniform solid medium. The method is intended to reduce the processing time (CPU time) needed to compute 3D heat diffusion using a TBEM formulation.

In the proposed formulation each inclusion is modelled individually and successively. The first crack is submitted to an incident heat field and produces a disturbance. Each one of the cracks analyzed next is reached by a heat field generated by the previous one, which is seen as an incident field. The iterative process is stopped when the heat field disturbance generated by each inclusion is negligible. The final solution is the sum of all the contributions (disturbances in the heat field).

Performance of the iterative approach proposed in this study is evaluated by comparing results generated using the full 3D TBEM and using the iterative model, in terms of temperature results, CPU time required for a given frequency, as well as number of iterations. The applicability of the proposed method is illustrated via a numerical example of heat field in time domain computation.

1. Introduction

Numerical modelling of heat diffusion in media with embedded inclusions has been used to evaluate the influence that defects have on temperature distribution and heat fluxes inside the domain, which has proved to be useful in several areas. In particular, numerical models can be useful in establishing the limits of the effectiveness of using infrared-thermography (IRT) in defect detection studies, as stated by Maldague [1]. In such studies, the numerical modelling of inclusions with different geometries can indicate defect detectability and determine the optimum observation time window of the best subsurface defect visibility without the expense of making and testing specimens. Solving transient heat transfer problems is useful for the detection and quantitative characterization of the properties of subsurface defects, but the 3D nature of subsurface defects, plus the need to simulate heat transfer and diffusion phenomena in transient regime, presents challenges for researchers [2,3].

Conventionally, numerical modelling requires either the discretization of the domain of the problem, as in finite elements methods (FEM) [4] and finite differences methods (FDM) [5,6], or the discretization of the boundary in the case of the boundary element method (BEM) [7,8]. Researchers have recently been developing meshless formulations which require neither domain nor boundary discretization, such as the method of fundamental solutions (MFS) [9]. Methods based on domain discretization are however better suited to deal with a bounded domain, since they require its full discretization. Furthermore, in the case of multiple inclusions the space between them also needs to be discretized, which requires special attention. This becomes unfeasible if there is a very large number of inclusions embedded in an unbounded domain.

Of the available numerical methods for homogeneous unbounded or semi-infinite systems modelling, the BEM is the technique that automatically satisfies far field conditions and therefore only requires the discretization of the boundaries of the inclusions. FDM and FEM techniques lead to sparse systems of equations, while the BEM results in fully populated systems of equations. Another drawback of the BEM is that it can only be applied to more general geometries and media when the relevant fundamental solutions are known. In addition, it is well known that when using the BEM the boundary integrals may become singular or nearly singular, depending on the distance between the source point and the node being integrated. Also, when the thickness of the heterogeneity being modelled tends towards zero, as in the case of delaminations, cracks or thin defects, the conventional direct BEM degenerates and becomes inaccurate, and is no longer a valid basis for numerical modelling. Among the

* Corresponding author. Tel.: +351 910304560.
E-mail addresses: cserra@itecons.uc.pt, inaserra@gmail.com (C. Serra), tadeu@dec.uc.pt (A. Tadeu), nasimoes@dec.uc.pt (N. Simões).

0955-7997/$ - see front matter © 2014 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.enganabound.2013.12.010

techniques that have been proposed to overcome this is the dual boundary element method (DBEM) or the normal-derivative integral equation (TBEM), which leads to hypersingular integrals. The correct integration of singular and hypersingular integrals is one of the big challenges of these techniques. A number of approaches have been proposed to deal with hypersingular integrals that arise in DBEM [10]. Solutions for specific 2D problems can be found in Cruse [11], Sládek and Sládek [12], Prosper [13], Prosper and Kausel [14], Amado Mendes and Tadeu [15]. Tadeu et al. [17] described an analytical evaluation of the singular and hypersingular integrals that appear in 3D boundary element formulations for heat diffusion, in the frequency domain. More recently, Matsumoto et al. [16] presented a solution for exterior acoustic problems governed by the Helmholtz equation using the normal derivative Burton–Miller formulation which leads to hypersingular integrals, by using constant triangular elements to discretize the boundary.

Nevertheless, the BEM is still regarded as one of the most suitable tools for modelling 3D heat diffusion generated by heat sources in an unbounded spatially uniform solid medium. However, these simulations are usually associated with a high computational effort. Various BEM formulations aimed at reducing it have been proposed. Ma et al. [18] used a BEM formulation to study transient heat conduction in 3D solids with fiber inclusions. Jablonski [19] proposed the analytical evaluation of the BEM surface integrals by means of 3D Laplace and Poisson equations. Qin et al. [20] implemented changes to the conventional distance transformation technique to evaluate nearly singular integrals on 3D boundary elements, including planar and curved surface elements and very irregular elements of slender shape.

Iterative solvers or multi-domain methods have also been suggested by authors to solve/avoid systems in 3D BEM problems with very large meshes. As described by Škerget and Rek [21] and Ramšak et al. [22], by using a multi-domain method the system matrix becomes sparse, thus considerably decreasing the amount of memory needed. By analyzing subdomains separately, where independent discretization can be considered for each subdomain and suitable solvers can be used for their individual systems of equations, smaller and better conditioned systems of equations can be obtained. One disadvantage of 3D multi-domain BEM stems from the difficulties of applying interface conditions between subdomains with a highly sparse system matrix. To overcome this, Ramšak and Škerget have extended their previous work using discretization of mixed elements [21,22] to 3D problems, and proposed a 3D BEM formulation for very large meshes using a multi-domain method where each element is itself a subdomain [23].

Valente and Pina [24] have explored iterative techniques based on conjugate gradient type methods as an alternative to the direct solution techniques for large scale three-dimensional problems. They concluded that these methods are competitive for BEM algebraic systems of equations, especially if used with an appropriate preconditioner [25]. Researchers such as Marburg and Schneider [26], Ylä-Oijala and Järvenpää [27] and Alia et al. [28] have proposed iterative approaches for acoustics problems.

In this paper we describe an iterative approach to simulate 3D heat diffusion in the presence of multi-inclusions using the BEM formulated in the frequency domain. Null thickness inclusions are dealt with by means of a normal-derivative equation formulation. Because only one inclusion is solved at each step, the matrix storage requirements are reduced, as a smaller system of equations is generated. In the first iteration each inclusion is solved considering the prescribed boundary conditions and disregarding the other inclusions. In each of the subsequent iterations only the heat field disturbance generated by the inclusions modelled previously (viewed as an incident field) is considered. Since the coefficient matrixes remain the same throughout the process, the systems of equations are only solved during the first iteration. The iterative process is stopped when the new heat field generated by each inclusion is sufficiently small. This is achieved by defining a convergence criterion which consists of comparing the responses obtained in specific receivers from each iteration. Performance of the proposed iterative 3D TBEM is studied by comparing the results generated by the full 3D normal-derivative equation formulation with those obtained using the iterative model. The number of iterations and the CPU time taken to compute the numerical responses are used to evaluate the computational efficiency of the proposed iterative formulation.

In the sections that follow, first the problem is defined and the iterative boundary element formulation used is presented (an iterative approach to the normal-derivative integral equations—3D TBEM—formulated in the frequency domain). Analytical solutions are used to solve the hypersingular integrals that appear in the 3D TBEM formulation when the element being integrated is the loaded element. The performance of the proposed iterative method is studied. The method employed to obtain time-domain responses from frequency-domain calculations is also described. Finally, a numerical application is presented to illustrate the applicability and usefulness of the proposed approach.

2. Numerical formulation

2.1. Problem definition

In this paper the three-dimensional (3D) heat diffusion generated by a point heat source in the presence of 3D cracks is modelled. The cracks are embedded in an unbounded spatially uniform solid medium of density ρ, thermal conductivity λ and specific heat c. To illustrate the numerical formulation used, consider two 3D cracks buried in a uniform medium, with surfaces S_1 and S_2, subjected to a point heat source O, as shown in Fig. 1. The cracks are assumed to be of null thickness, and null heat fluxes are prescribed along their surfaces.

The harmonic point heat source O placed at $\boldsymbol{x}_s=(x_s, y_s, z_s)$ generates an incident heat field in $x=(x,y,z)$ which is expressed by

$$T_{inc}(\boldsymbol{x}, \boldsymbol{x}_s, \omega)=\frac{P\, e^{-i\sqrt{-(i\omega/K)}r_0}}{2\lambda r_0}, \quad (1)$$

in which T_{inc} is the incident heat field at $\boldsymbol{x}$ when the point heat source is located at $\boldsymbol{x}_s$ and oscillates at a frequency of ω, K is the thermal diffusivity defined by $\lambda/\rho c$, $i=\sqrt{-1}$, $r_0=\sqrt{(x-x_s)^2+(y-y_s)^2+(z-z_s)^2}$ and P is the amplitude of the heat source.

2.2. The iterative 3D normal derivative integral equation (iterative 3D TBEM)

This section describes the iterative 3D TBEM formulation used to simulate 3D heat diffusion in the presence of multiple cracks, generated by a point heat source.

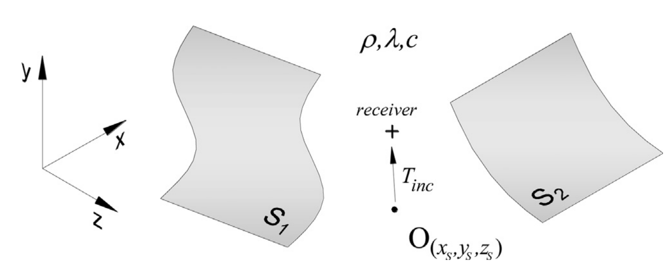

Fig. 1. 3D view of the geometry of the problem.