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Revisited mixed-value method via symmetric BEM in the substructuring approach

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ABSTRACT

Within the Symmetric Boundary Element Method, the mixed-value analysis is re-formulated. This analysis method contemplates the subdivision of the body into substructures having interface kinematical and mechanical quantities. For each substructure an elasticity equation, connecting weighted displacements and tractions to nodal displacements and forces of the same interface boundary and to external action vector, is introduced. The assembly of the substructures is performed through both the strong and weak regularity conditions of the displacements and tractions. We obtain the solving equations where the compatibility and the equilibrium are guaranteed in the domain Ω for the use of the fundamental solution and at the interface nodes for the strong regularity conditions imposed, whereas the previous quantities are respected in weighted form along the interface boundaries. The mixed-value method leads to a better solution than those obtained through the displacement method of the Symmetric Boundary Element Method, if compared with the analytical solution.

By using the Karnak.sGbem program, developed with other researchers and updated through the implementation of the present method, some examples are made which show the advantages related to the computational aspects and to the convergence of the numerical response.

1. Introduction

In all fields of the applied mechanics and engineering the use of analysis methods able to provide solutions closer to the real response, by using a restricted number of variables, is desirable. Among these, the Symmetric Boundary Element Method (SBEM) is used, which through its characteristics allows one to reach these goals. Indeed it works using elements having large dimensions characterized only by boundary quantities and uses fundamental solutions, thus making it possible to have considerable accuracy in the response due to the satisfaction of the equilibrium and compatibility in the domain. The statical and kinematical conditions are only imposed in weighted form on the boundary and the quality of the solution depends on the boundary discretization and on the variable modelling.

The analysis regarding bodies having zone-wise different physical characteristics requires particular strategies and only recently were some difficulties overcome within the substructuring process. This idea of subdividing the bodies into macro-elements having different physical properties was introduced by Maier et al. [1]. Subsequently, Gray and Paulino [2] utilized substructuring in potential problems. Layton et al. [3] proposed a formulation which divides the body into macro-zones, each of which is governed by boundary quantities, and which after a condensation process gives rise to a system having only a few asymmetric blocks. In a subsequent paper the same authors [4] presented a truly symmetric method for a two-zone elastic problem characterized by interface and non-interface unknowns. Cen et al. [5] developed a multi-zone approach, where a non-symmetric mixed-variable solving system, having the unknown vector only with interface nodal mechanical and kinematical quantities, was considered within the cohesive fracture case. Pérez-Gavilán and Aliabadi [6] proposed an approach only having kinematical variables and showed the non-uniqueness of the solution.

Panzeca et al. [7–11] dealt with the substructuring problem among substructures proceeding in accordance with a variational formulation proposed by Maier and Polizzotto [12], Polizzotto [13,14] for a single body, reaching an approach having only interface unknowns among substructures. This strategy led to different approaches defined as a mixed-value method (Panzeca et al. [7,8]) and as a displacement method (Panzeca et al. [10,11]), both characterized by the employment of strong (nodal quantities) and weak (weighted

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quantities) regularity conditions at the interface and by substantial variable condensation. In the displacement method, implemented in the Karnak.sGbem code (Cucco et al. [15]), the authors distinguished the substructures into macro-elements having generally large dimensions and elastic materials and bem-elements having small dimensions where, in the presence of physically nonlinear behaviour, the possible plastic strains can be evaluated (Zito et al. [16]).

Kallivokas et al. [17] proposed an energy-based variational framework for the solution of interior problems in multiply-connected domains comprising multiple piecewise homogeneous subdomains, using exclusively boundary integral equations, and provided a unified variational setting that leads to symmetric Galerkin boundary element formulations in terms of Dirichlet-type variables only. The presence of Neumann-type boundary unknowns is suppressed through a condensation process at the subdomain level.

Vodicka et al. [18,19] proposed an approach, based on a variational principle, to solve domain decomposition problems by the symmetric Galerkin boundary element method (DDBVP) between two substructures. Weak coupling conditions of equilibrium and compatibility at the interface are obtained from the critical point conditions of the energy functional. A natural property of this approach is its capability to deal with nonconforming discretization along straight and curved interfaces, thus allowing independent meshing of subdomains to be performed.

Bonnet G. et al. [20] utilized the symmetric BEM to find the boundary stiffness matrix characterizing the response of the discrete boundary to given Dirichlet boundary conditions within the coupling between BEM and FEM.

The substructuring process was also dealt with within the collocation BEM by Lu and Wu [21], who significantly reduce the size of the final matrix; Sfantos and Aliabadi [22], who present a sensitivity formulation for contact problems having as unknowns the normal gaps; Gao et al. [23,24], who in the final system of equations obtain only interface displacements as unknowns; Phan et al. [25], who deal with a general type of displacement and traction conditions at the interface; Bui et al. [26], who simplify the assembly of the equations arising from the BEM sub-domain methods reducing the size of the system matrix. Other papers regarding substructuring within the collocation BEM are included in the related references.

Within symmetric BEM, in this paper, on the basis of a strategy performed by Panzeca et. al. [10], the mixed-value method, proposed by Panzeca et al. [7,8], is re-formulated through (a) the subdivision of the body by substructures having any shape and dimension, (b) the writing of an elasticity equation for each substructure connecting weighted quantities (displacements and tractions) to nodal quantities (displacements and forces) and to the load vector, (c) the use of the equation system through the writing of equilibrium and compatibility strong forms at the interface nodes and through the related weak forms involving weighted (or generalized) displacements and tractions at the interface boundary elements. Since the discretization is made at the beginning of the analysis, in the present state of research the interfaces between adjacent substructures are discretized using conforming meshes with straight boundary elements. The introduction of nonconforming meshes to deal with particular problems like contact will be the object of a new paper.

The proposed mixed-value method was recently implemented through additional procedures within the Karnak.sGbem program, already using the displacement method.

Some examples are studied to check the quality of the solution through the comparison of the results with those obtained with the displacement method based on the same formulation, and, when it is possible, with a closed form response

2. Elastic analysis

A plane bidimensional body of domain Ω and boundary Γ, referred to a Cartesian orthogonal co-ordinate system (O,x,y), is subjected to quasi-static actions in its plane (Fig. 1a): forces $\bar{\mathbf{f}}$ at the portion Γ_2 of the free boundary, displacements $\bar{\mathbf{u}}$ at the portion $\Gamma_1=\Gamma\backslash\Gamma_2$ of the constrained boundary, and body forces $\bar{\mathbf{b}}$ and volumetric (thermal and plastic) strains $\bar{\vartheta}$ in Ω.

In this body the physical and geometrical characteristics are step-wise variables. One wants to obtain the elastic response to external actions in terms of displacements $\mathbf{u}$ on Γ_2, reactions $\mathbf{f}$ on Γ_1, displacements, strains and stresses in Ω by using the SBEM, through the mixed-value formulation.

For this purpose an appropriate subdivision of the domain into substructures is made, in order to have the physical and geometrical characteristics, constant in each substructure. This subdivision involves the introduction of interface boundaries Γ_0 and the related rise of traction $\mathbf{t}_0$ and displacement $\mathbf{u}_0$ distribution vectors (Fig. 1b). For each of the substructures, the Somigliana Identities of the displacements $\mathbf{u}$ and of the tractions $\mathbf{t}$ in Ω can be written as functions of known and unknown actions of the boundary Γ and of the domain Ω.

The latter are written in compact form as follows:

$$\mathbf{u}=\mathbf{u}\ [\mathbf{f}]+\mathbf{u}\ [\mathbf{v}]+\bar{\mathbf{u}}\ [\bar{\mathbf{b}},\bar{\vartheta}] \tag{1a}$$

$$\mathbf{t}=\mathbf{t}\ [\mathbf{f}]+\mathbf{t}\ [\mathbf{v}]+\bar{\mathbf{t}}\ [\bar{\mathbf{b}},\bar{\vartheta}] \tag{1b}$$

where $\mathbf{v}=\mathbf{0}-\mathbf{u}$ is the jump in the displacement between those of the boundary of the complementary domain (null by definition) and those of the real one. In the previous equations, the following symbolic form was employed like in Polizzotto [13]:

$$\mathbf{u}\ [\mathbf{f}]=\int_\Gamma \mathbf{G}_{uu}\,\mathbf{f}\ d\Gamma \quad \mathbf{u}\ [\mathbf{v}]=\int_\Gamma \mathbf{G}_{ut}\,(\mathbf{v})\ d\Gamma \tag{2a, b}$$

$$\mathbf{t}\ [\mathbf{f}]=\int_\Gamma \mathbf{G}_{tu}\,\mathbf{f}\ d\Gamma \quad \mathbf{t}\ [\mathbf{v}]=\int_\Gamma \mathbf{G}_{tt}\,(\mathbf{v})\ d\Gamma \tag{2c, d}$$

$$\hat{\mathbf{u}}\ [\bar{\mathbf{b}},\bar{\vartheta}]=\int_\Omega \mathbf{G}_{uu}\,\bar{\mathbf{b}}\ d\Omega+\int_\Omega \mathbf{G}_{u\sigma}\,\bar{\vartheta}\ d\Omega\hat{\mathbf{t}} \quad [\bar{\mathbf{b}},\bar{\vartheta}]=\int_\Omega \mathbf{G}_{tu}\,\bar{\mathbf{b}}\ d\Omega+\int_\Omega \mathbf{G}_{t\sigma}\,\bar{\vartheta}\ d\Omega \tag{2e, f}$$