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Extending MLPG primitive variable-based method for implementation in fluid flow and natural, forced and mixed convection heat transfer

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ABSTRACT

The purpose of the current study is to empower the MLPG primitive variable-based method using the characteristic-based split (CBS) scheme to solve the laminar fluid flow and natural, forced, and mixed convection heat transfer at, respectively, higher Rayleigh, Reynolds and Peclet, and Reynolds and Grashof numbers than those that the MLPG approach has ever solved. In this work, the CBS scheme with unity test function is employed for discretization and the moving least square (MLS) method is used for interpolation. As some test cases, natural convection within a square cavity, forced convection by fluid flow over a bundle of tubes, and mixed convection within a lid-driven square cavity are solved by the proposed method. For verifications, the obtained results are compared with those of the conventional numerical methods in the literature. Being entirely meshless, strong in nature, and able to give accurate and stable results for the broadest range of laminar fluid flow involving any of the three modes of convection heat transfer, the proposed method shows to be a flexible and reliable technique which can replace many available meshfree methods in the literature.

1. Introduction

During the past decade, a number of meshless, also called, meshfree techniques have been developed by researchers to alleviate the mesh generation hardships, high computational costs, and remeshing procedures especially in three-dimensional problems. Among the available meshfree techniques are smooth particle hydrodynamics (SPH) method [1], diffuse element method (DEM) [2], partition of unity method (PUM) [3], meshless local Petrov–Galerkin (MLPG) method [4], local boundary integral equation (LBIE) method [5], and others. Most meshless methods concentrate on solid mechanics [6–8], heat conduction [9–15], p-Laplace equation [16], sine-Gordon equation [17], Eikonal equation [18], and magnetohydrodynamic (MHD) flow [19] areas. However, there are a few implementations of these meshfree techniques in the fluid flow and convection heat transfer arenas.

Sadat and Couturier [20] used the diffuse approximation method (DAM) for solving two-dimensional incompressible laminar thermal fluid flows. The projection algorithm was used to treat the velocity–pressure coupling. Their work considered the natural convection heat transfer in a square cavity for up to $Ra = 10^8$ and in an eccentric circular annulus for up to $Ra = 10^7$. In another work, Sophy et al. [21] used the same approach to solve the three-dimensional differentially heated cubic cavity for up to $Ra = 10^6$.

Kosec and Šarler [22] applied the meshfree local radial basis function collocation method (RBFCM) to solve the natural convection heat transfer in a square cavity for up to $Ra = 10^8$ using the primitive variable formulation. However, in their study they reported that, the pressure correction calculation method may not be efficient enough when working with finer grids (200×200 or finer) and for higher Rayleigh numbers (higher than 5×10^7), hence, they recommended further investigations on those topics. Szewc et al. [23] introduced a new variant for the smoothed particle hydrodynamic (SPH) simulations for two natural convection heat transfer problems. They solved a Rayleigh–Taylor instability problem and a natural convection in a square cavity problem for up to $Ra = 10^5$. Wu and Liu [24] adopted the local radial point interpolation method (LRPIM) to simulate the two-dimensional natural convection cases. They used the vorticity-stream function form of the Navier–Stokes equations and solved the natural convection in a square cavity for up to $Ra = 10^5$ and in a concentric circular annulus for up to $Ra = 10^4$.

Liu et al. [25] used the meshfree weak-strong (MWS) form method for solving the Navier–Stokes equations in the form of stream function–vorticity formulation. The heat transfer test case in this study was the natural convection in a square cavity for up to $Ra = 10^5$.

Abbreviations: DAM, diffuse approximation method; DSC, discrete singular convolution; FDM, finite difference method; FEM, finite element method; FVM, finite volume method; MLPG-PV, meshless local Petrov–Galerkin primitive variable.

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Nomenclature

c_p	specific heat
D	diameter
F	body force
g	gravitational acceleration
h	minimum distance between the discretized point to the boundary of the quadrature domain
Gr	Grashof number
k	thermal conductivity
L	characteristic length scale, length
n_i, n_k	components of outward normal vector
Nu	Nusselt number
P	pressure
Pe	Peclet number
Pr	Prandtl number
Ra	Rayleigh number
Re	Reynolds number
Ri	Richardson number ($\mathrm{Ri}=\mathrm{Gr}/\mathrm{Re}^2$)
S_L	longitudinal pitch
S_T	transverse pitch
T	temperature
t	time
u	velocity in x direction
u_a	characteristic velocity, velocity
u_i	velocity components
v	velocity in y direction
W	weighting function
x_i	coordinate components
x, y	Cartesian coordinates

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
Γ	global boundary ($\Gamma_u \cup \Gamma_t$)
Γ_s	boundary of Ω_s
Γ_t	natural boundary
Γ_u	essential boundary
Γ_{si}	internal boundary of Ω_s
Γ_{st}	intersection of Γ_s and Γ_t
Γ_{su}	intersection of Γ_s and Γ_u
θ_1, θ_2	constant parameters
ν	kinematic viscosity
ρ	density
ψ	stream function
Ω_s	local sub-domain

superscripts

$-$	with dimension
$\sim$	intermediate
n	time level

subscripts

c	cold, convection velocity
du	velocity diffusion
dT	temperature diffusion
h	hot
i, j, k	notation tensor $(1, 2)$
max	maximum
mid	value at the midpoint of the cavity

In recent years, the MLPG method [4,26], known as a, truly totally meshless technique, has considered a few fluid flow and heat convection problems. Wu et al. [27] applied the MLPG method to simulate the incompressible fluid flow in terms of the stream function–vorticity formulation within an irregular domain with scattered nodal distribution. They solved the natural convection heat transfer in the annulus of two concentric circular cylinders for Rayleigh numbers of up to 10^4 and also the natural convection in the annulus of a concentric circular inner and square outer cylinder for Rayleigh numbers of up to 10^6.

Arefmanesh et al. [28] employed the MLPG method to solve the Navier–Stokes equations in terms of the stream function–vorticity formulation. The heat transfer test cases in this study were a non-isothermal
lid-driven cavity fluid flow for $\mathrm{Re}=100$ and $\mathrm{P}e=50$ and a non-isothermal fluid flow over an obstacle for $\mathrm{Re}=30$ and $\mathrm{P}e=3$. In an another study, Arefmanesh et al. [29] employed the MLPG method for solving the natural convection heat transfer in terms of the stream function–vorticity formulation in a cavity with wavy side walls for up to $\mathrm{Ra}=10^6$. Through another study, Arefmanesh et al. [30] employed the MLPG stream function–vorticity based method in simulating nanofluid flow mixed convection heat transfer in a wavy wall cavity for different Richardson numbers and nanoparticles volume fractions. The maximum Reynolds and Grashof numbers that they used in their work were 1000 and 10^4, respectively. Nikfar and Mahmoodi [31] applied the MLPG method using the stream function–vorticity formulation to analyze the natural convection of nonofluids in a cavity with wavy side walls for different Rayleigh numbers of up to 10^6.

As the above mentioned studies show, the MLPG vorticity function-based method was utilized for just a few fluid flow and heat convection problems. However, the stream function–vorticity method, as it is known, is not considered as a flexible and strong method and is looked upon as a complicated method for implementation in three-dimensional problems and in most of two-dimensional complex geometries.

Wu et al. [32] extended the MLPG method to solve incompressible fluid flow problems based on the primitive variable formulation. In their study, the streamline upwind Petrov–Galerkin (SUPG) method was applied to overcome the oscillations in their convection-dominated problems. The heat transfer test case in their study was natural convection in a square cavity for Rayleigh numbers of up to 10^4. They concluded that, their method for low Reynolds and Rayleigh numbers was good, but stated that, further investigations were needed to improve the precision of convection-dominated problems at high Reynolds numbers.

Based on a thorough literature search conducted, there is quite a dearth of meshfree numerical work in convection heat transfer arena. In fact, the very little available meshless works performed are limited to a very few particular cases or a few low Rayleigh, Reynolds, Grashof, and Peclet numbers convection heat transfer problems. In this present