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Harmonic analysis of shear deformable orthotropic cracked plates using the Boundary Element Method

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ABSTRACT

In this work, the modal and harmonic analysis of orthotropic shear deformable cracked plates using a direct time-domain Boundary Element Method formulation based on the elastostatic fundamental solution of the problem is presented. The Radial Integration Method was used for the treatment of domain integrals involving distributed domain applied loads and those related with inertial mass forces. Numerical examples are presented to demonstrate the efficiency and accuracy of the proposed formulation.

1. Introduction

The Boundary Element Method (BEM) is an attractive numerical alternative to treat fracture problems in shear deformable isotropic plates, mainly due to its ability to model continuously high stress gradients (like those found near to the crack tip) without the need of domain discretization. Successful formulations based on the Dual Boundary Element Method were developed for the analysis of cracked shear deformable plates and shells under static loads [1].

For plate bending dynamic analysis, BEM has emerged as an accurate and efficient numerical method [2]. Boundary element solutions for dynamic plate problems are usually obtained by using three basic approaches: formulations based on elastodynamic fundamental solutions, formulations based on Laplace or Fourier transformations of the elastodynamic fundamental solutions [11–13] and direct formulations based on elastostatic fundamental solutions [18]. The simplicity of the real valuated static fundamental solution is an advantage if compared to the complicated complex valuated elastodynamic fundamental solution. In this approach, boundary element formulations create domain integrals due to the presence of inertial terms. In order to treat these domain integrals the Dual Reciprocity Method (DRM) [16] and the Radial Integration Method (RIM) [14,15] have been used.

In the last decade, few works have been reported in the literature related to static and dynamic analysis of cracked orthotropic plates. To the author's knowledge, none of them applied to the dynamic analysis of crack problems in shear deformable orthotropic plates. In [20], a displacement discontinuity formulation for the BEM analysis of crack orthotropic shear deformable plates under static loads is presented. In [3,4] the dual boundary element method is applied to the analysis of anisotropic cracked plates under static in-plane loads. The dynamic analysis of cracked anisotropic plates using a direct time-domain formulation and formulations based on the Laplace transformation is presented in [5–7] where the Dual Boundary Element Method and the Multi-Domain Method were used to fracture modeling. In [8] the Radial Integration Method was applied for the analysis of symmetric laminated thin plates considering transverse static distributed loads. This work was extended to the analysis of laminated thin plates under dynamic loads as reported in [9]. In [10] this formulation was applied to the modal analysis of thin symmetrical laminated plates.

In this paper, the time-domain boundary element method formulation presented in [18] is applied to modal and harmonic analysis of cracked orthotropic shear deformable plates. The fundamental solutions for elastostatics developed by Wang [19] are used and the inertia terms are treated as body forces. The RIM is used to transform the domain integrals into boundary integrals. Cracked plates were modeled using the multi-domain technique.

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Numerical results show a good agreement when compared with those obtained using the Finite Element Method.

2. Plate bending equations

Consider an elastic orthotropic plate of uniform thickness h, mass density ρ, occupying an area Ω in the x_1x_2 plane, bounded by the contour $\Gamma=\Gamma_w\bigcup\Gamma_p$ with $\Gamma=\Gamma_w\bigcap\Gamma_p\equiv 0$. The bending response for the plate was modelled using the First Order Shear Deformation Theory. The relations between the generalized displacements and strains are [1]:

$$2\chi_{\alpha\beta}=w_{\alpha,\beta}+w_{\beta,\alpha}$$

$$\psi_\alpha=w_\alpha+w_{3,\alpha} \quad (1)$$

where $\chi_{\alpha\beta}$ is the curvature tensor and ψ_α are the transversal shear strains. Indicial notation is used throughout this work. Greek indices vary from 1 to 2 and Latin indices take values from 1 to 3. Einstein's summation convention is used unless otherwise indicated. In these equations, w_α represents rotations with respect to x_1 and x_2 axes and w_3 represents transversal deflection.

The equilibrium equations for an infinitesimal shear deformable plate under dynamical distributed loading q_i are given by [18]

$$M_{\alpha\beta,\beta}-Q_\alpha+q_\alpha=\Lambda_{\alpha\beta}w_{\beta,tt} \quad (2)$$

$$Q_{\alpha,\alpha}+q_3=\Lambda_{33}w_{3,tt} \quad (3)$$

where $M_{\alpha\beta}$ and Q_α are the resultant tensor moment and the normal shear vector, respectively. $w_{\alpha,tt}$ denotes angular accelerations with respect to x_1 and x_2 axes, respectively, $w_{3,tt}$ represents the transverse acceleration. q_α are distributed moments and $q_3=p$ represents distributed pressure. Λ_{ik} is a tensor defined as: $\Lambda_{\alpha\beta}=1/12\rho h^3\delta_{\alpha\beta}$, $\Lambda_{33}=\rho h$ and $\Lambda_{\alpha 3}=\Lambda_{3\alpha}=0$.

For an orthotropic plate, the resultant tensor moment $M_{\alpha\beta}$ and the normal shear vector Q_α are [19]:

$$M_{\alpha\beta}=D_{\alpha\beta}(w_{\alpha,\beta}+w_{\beta,\alpha})+C_{\alpha\beta}w_{\gamma,\gamma} \quad (4)$$

$$Q_\alpha=C_\alpha\psi_\alpha \quad (5)$$

where elastic constants $D_{\alpha\beta}$, $C_{\alpha\beta}$ and C_α are given by

$$C_{12}=C_{21}=0,\quad C_{11}=D_1\nu_{21},\quad C_{22}=D_2\nu_{12}$$

$$D_1=\frac{E_1h^3}{12(1-\mu_{12}\mu_{21})},\quad D_2=\frac{E_2h^3}{12(1-\mu_{12}\mu_{21})},\quad D_k=\frac{G_{12}h^3}{12}$$

$$D_1\nu_{21}=D_2\nu_{12},\quad C_1=G_{31}kh,\quad C_2=G_{32}Kh \quad (6)$$

In these expressions, E_α represents elastic modulus, $G_{i\beta}$ are shear modulus, $\nu_{\alpha\beta}$ and $K=5/6$ is the shear correction factor.

Substituting Eqs. (1) and (5) into equilibrium Eqs. (3), we obtain the following differential equations using generalized displacements as basic unknowns:

$$L_{ik}w_k+q_i=\Lambda_{ik}w_{k,tt} \quad (7)$$

Differential operator L_{ik} is defined as:

$$L_{11}=D_1\frac{\partial^2}{\partial x_1^2}+D_k\frac{\partial^2}{\partial x_2^2}-C_1,\quad L_{22}=D_k\frac{\partial^2}{\partial x_1^2}+D_2\frac{\partial^2}{\partial x_2^2}-C_2,$$

$$L_{12}=L_{21}=(D_1\mu_{21}+D_k)\frac{\partial^2}{\partial x_1\partial x_2},\quad L_{13}=-L_{31}=-C_1\frac{\partial}{\partial x_1},$$

$$L_{23}=-L_{32}=-C_2\frac{\partial}{\partial x_2},\quad L_{33}=C_1\frac{\partial^2}{\partial x_1^2}+C_2\frac{\partial^2}{\partial x_2^2} \quad (8)$$

The generalised boundary conditions for these equations are given by:

$$w_k(\mathbf{x},t)=\overline{w}_k \quad \forall\ \mathbf{x}\in\Gamma_w$$

$$p_\alpha(\mathbf{x},t)=M_{\alpha\beta}n_\beta=\overline{p}_\alpha \quad \forall\ \mathbf{x}\in\Gamma_p$$

$$p_3(\mathbf{x},t)=Q_\alpha n_\alpha=\overline{p}_3 \quad \forall\ \mathbf{x}\in\Gamma_p \quad (9)$$

where p_α represents in-plane moments and p_3 is the shear force. $\overline{w}_k$ and $\overline{p}_k$ are generalised displacement and tractions imposed at Γ_w and Γ_p, respectively. In a similar way, initial conditions are given by:

$$w_k(\mathbf{x},0)=\overline{w}_k^0 \quad \forall\ \mathbf{x}\in\Omega,\ t\in[0,t_{max}]$$

$$\dot{w}_k(\mathbf{x},0)=\overline{\dot{w}}_k^0 \quad \forall\ \mathbf{x}\in\Omega,\ t\in[0,t_{max}] \quad (10)$$

3. Boundary integral formulation

The derivation of the integral formulation for Eqs. (7) is based on the application of the BEM to the shear deformable plate theory as presented in [1], where integral representations related to the governing equations for bending and transverse shear stress resultants are derived by using the weighted residual method, and making use of Green's identity. Thus, the integral formulation for these equations (considering $q_\alpha=0$ and $q_3=p$) is given by

$$c_{ik}w_k(\mathbf{x}',t)+\int_\Gamma P^*_{ik}(\mathbf{x}',\mathbf{x})w_k(\mathbf{x},t)\,d\Gamma=\int_\Gamma W^*_{ik}(\mathbf{x}',\mathbf{x})p_k(\mathbf{x},t)\,d\Gamma$$
$$+\int_\Omega W^*_{i3}(\mathbf{x}',\mathbf{x})p(\mathbf{x},t)\,d\Omega-\int_\Omega W^*_{ik}(\mathbf{x}',\mathbf{x})\Lambda_{ik}w_{k,tt}(\mathbf{x},t)\,d\Omega \quad (11)$$

where W^*_{ik} and P^*_{ik} are fundamental solutions of the orthotropic shear deformable plates as presented in [19]; $c_{ij}(\mathbf{x}')$ are the jump terms arising from the terms of $O(1/r)$ in the kernel P^*_{ij}. Eq. (11) represents three integral equations, two (i=1,2) for rotations and one for deflection.

4. Transformation of domain integrals

In order to transform domain integrals related with distributed pressure and inertial terms into boundary integrals, the Radial Integration Method—RIM—was used as presented in [10] where body forces are approximated with the use of radial basis functions.

Consider the domain term $p(t)$ in Eq. (11) approximated over the domain as a sum of the M products between radial basis functions f_m and unknown coefficients $\gamma^m(t)$, that is:

$$p(t)\approx\sum_{m=1}^{M}f^m\gamma^m(t) \quad (12)$$

The second domain integral of Eq. (11) in the right-hand side can be written as

$$\int_\Omega W^*_{i3}p(t)\,d\Omega=\sum_{m=1}^{M}\gamma^m(t)\int_\Omega W^*_{i3}(\mathbf{x}',\mathbf{x})f^m\,d\Omega$$
$$=\sum_{m=1}^{M}\gamma^m(t)\int_\theta\int_0^r W^*_{i3}(\mathbf{x}',\mathbf{x})f^m\rho\,d\rho\,d\theta \quad (13)$$

where r is the value of ρ in a point of the boundary Γ as shown in Fig. 1. Defining E^m_{i3} as the following integral:

$$E^m_{i3}(\mathbf{x}')=\int_0^r W^*_{i3}(\mathbf{x}',\mathbf{x})f^m\rho\,d\rho \quad (14)$$