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# Analysis of elastic wave propagation in a functionally graded thick hollow cylinder using a hybrid mesh-free method

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ABSTRACT

In this paper, a hybrid mesh-free method based on generalized finite difference (GFD) and Newmark finite difference (NFD) methods is presented to calculate the velocity of elastic wave propagation in functionally graded materials (FGMs). The physical domain to be considered is a thick hollow cylinder made of functionally graded material in which mechanical properties are graded in the radial direction only. A power-law variation of the volume fractions of the two constituents is assumed for mechanical property variation. The cylinder is excited by shock loading to obtain the time history of the radial displacement. The velocity of elastic wave propagation in functionally graded cylinder is calculated from periodic behavior of the radial displacement in time domain. The effects of various grading patterns and various constitutive mechanical properties on the velocity of elastic wave propagation in functionally graded cylinders are studied in detail. Numerical results demonstrate the efficiency of the proposed method in simulating the wave propagation in FGMs.



## 1. Introduction

One of the most important points in designing procedure of functionally graded materials (FGMs) is the analysis of functionally graded structures under dynamic, in particular, shock loadings. The wave propagation in functionally graded structures is initiated by sudden mechanical or thermal loading or unloading. There are several solution techniques to solve the equations of motion for elastic wave propagation and coupled thermo-elasticity for finite speed thermal wave propagation in FGMs. A brief review of literature that forms a background for the present study is provided as follows.

Elastic wave propagation in functionally graded cylinders with and without piezo-electric layers was studied by Han et al. [1,2]. A rule of mixture, called as SBS method with multilayer modeling was used. One-dimensional elastic wave propagation in linear elastic materials as well as laminated composites consisting of linear elastic laminae was studied by Surana et al. [3] using the coupled space-time least-squares finite element processes in the *h*, *p*, *k* framework. They considered macro-interlamina physics which, in the presence of bi-material interfaces, requires the continuity and higher-order global differentiability of the axial force at the bi-material interface and thereby necessitating the discontinuity of the spatial derivative of the axial displacement at the interface. The transverse surface waves (Love waves) were analyzed in a piezoelectric half space of polarized ceramics carrying a FGM layer by Qian et al. [4]. The propagation behavior of Love waves was studied from the three-dimensional equations of linear piezoelectricity in their work. Hosseini et al. [5] studied the dynamic analysis of functionally graded cylinders subjected to mechanical shock loading. The mean velocity of radial stress wave propagation; natural frequency and dynamic behavior of FG cylinder were presented in their work using the Galerkin finite element (GFE) formulation with linear functionally graded elements for spatial variables and Newmark time integration scheme for time domain. Bin et al. [6] developed the application of the Legendre orthogonal polynomial series expansion approach to analyze the propagation of harmonic waves in imhomogeneous magneto-electro-elastic plates composed of piezoelectric and magneto-strictive materials. The different influences of the piezoelectricity and piezo-magnetism were studied in detail for various electric potential and magnetic potential distributions at different wave numbers.

Also, Hosseini et al. [7,8] studied heat wave propagation and coupled thermoelasticity without energy dissipation in functionally graded thick hollow cylinder of infinite and finite lengths based on Green–Naghdi theory. They used Galerkin finite element model to analyze the thermo-elastic wave propagation in functionally graded cylinders.

Thermo-mechanical vibration analysis of functionally graded (FG) beams and functionally graded sandwich (FGSW) beams were carried out by Pradhan and Murmu [9] considering the beam to be resting on Winkler foundation with two-parameter

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elastic foundation. The governing differential equations in their work were solved using the modified differential quadrature method (MDQM).

The dispersion behavior of waves in a functionally graded elastic plate was studied using laminate plate theory by Chen et al. [10]. They obtained scattering relations by considering the continuity conditions at the interfaces between any two layers and the boundary conditions at the upper and lower surfaces. Zhang and Batra [11] studied the propagation of elastic waves in FGMs using the modified smoothed particle hydrodynamics (MSPH). To control oscillations in the shock wave, an artificial viscosity was added to the hydrostatic pressure. An analytical method based on Bessel's functions was presented by Hosseini and Abolbashari [12], in which they studied the elastic wave propagation in functionally graded thick hollow cylinder. In their work, the functionally graded cylinder was excited by impact loading. The harmonic waves propagating between two certain points in inhomogeneous magneto-electro-elastic functionally graded spherical curved plates composed of piezoelectric and magneto-strictive materials were investigated with the Legendre orthogonal polynomial series expansion approach by Jiangong and Qiujuan [13].

Recently, the thermo-elastic wave propagation was stochastically studied using an hybrid numerical method (GFE, NFD and Monte Carlo simulation) for isotropic and functionally graded thick hollow cylinder by Hosseini and Shahabian [14,15]. The dispersion characteristics of wave propagation in a submerged piezoelectric transducer were studied by Chen and Bian [16] considering a functionally graded property. In their work, the effect of outer fluid was taken into account by imposing the exact continuity conditions at the fluid structure interface. The application of meshless local Petrov Galerkin (MLPG) method was used to study of the thermo-elastic wave propagation in functionally graded thick hollow cylinder using Green–Naghdi theory of coupled thermo-elasticity by Hosseini et al. [17]. Recently, Fu et al. [18] investigated the meshless boundary knot method (BKM) in conjunction with Kirchhoff transformation for heat conduction problems in nonlinear functionally graded material.

In this study, a hybrid mesh-free method based on generalized finite difference (GFD) and Newmark finite difference methods is developed to study the elastic wave propagation in functionally graded thick hollow cylinder. The functionally graded cylinder is excited by mechanical shock loading and time histories of radial displacement are obtained for various grading patterns of the material and also various constitutive mechanical properties. A mathematical approach is presented here to calculate the mean velocity of elastic wave propagation using the time history of radial displacement.

## 2. Governing equation of wave motion

Consider a thick hollow cylinder made of functionally graded materials (FGMs) with inner radius $r_{in}$ and outer radius $r_{out}$. To estimate the elastic wave propagation velocity, the wave motion equation should be solved in functionally graded cylinder. The governing equation of motion is

$$\frac{\partial \sigma_r}{\partial r}+\frac{\sigma_r-\sigma_\theta}{r}=\rho(r)\frac{\partial^2 u}{\partial t^2} \tag{1}$$

$$\sigma_r=c_1(r)\varepsilon_r+c_2(r)\varepsilon_\theta \tag{2}$$

$$\sigma_\theta=c_1(r)\varepsilon_\theta+c_2(r)\varepsilon_r \tag{3}$$

where $\sigma_r$, $\sigma_\theta$ are radial and hoop stresses, $\varepsilon_r$, $\varepsilon_\theta$ are radial and hoop strains, $r$ is radius, $\rho$ is density and $t$ is time.

$$c_1(r)=\frac{E(r)(1-\nu)}{(1+\nu)(1-2\nu)},\quad c_2(r)=\frac{E(r)\nu}{(1+\nu)(1-2\nu)} \tag{4}$$

The term "$\nu$" is Poisson's ratio, which is considered to be constant. The following nonlinear function in volume fraction form is considered to simulate the variation of mechanical properties through radial direction in functionally graded cylinder:

$$P(r)=(P_{out}-P_{in})\left(\frac{r-r_{in}}{r_{out}-r_{in}}\right)^n+P_{in} \tag{5}$$

where the terms $P_{in}$ and $P_{out}$ are mechanical properties of inner and surfaces of FGM cylinder, respectively. The modulus of elasticity and density are nonlinear functions of $r$ as follows:

$$E(r)=(E_{out}-E_{in})\left(\frac{r-r_{in}}{r_{out}-r_{in}}\right)^n+E_{in} \tag{6}$$

$$\rho(r)=(\rho_{out}-\rho_{in})\left(\frac{r-r_{in}}{r_{out}-r_{in}}\right)^n+\rho_{in} \tag{7}$$

The governing equation of motion can be rewritten in non-dimensional form using the following dimensionless parameters:

$$\bar{r}=\frac{r}{r_{out}},\quad \bar{t}=\frac{tV}{r_{out}},\quad \bar{u}=\frac{u}{r_{out}},\quad \bar{E}=\frac{E}{E_{in}},\quad \bar{\rho}=\frac{\rho}{\rho_{in}},\quad \bar{\sigma}_r=\frac{\sigma_r}{E_{in}},\quad \bar{\sigma}_\theta=\frac{\sigma_\theta}{E_{in}} \tag{8}$$

where $V$ is defined as

$$V=\sqrt{\frac{E_{in}}{\rho_{in}}} \tag{9}$$

Consequently, all equations in dimensionless form are

$$\frac{\partial \bar{\sigma}_r}{\partial \bar{r}}+\frac{\bar{\sigma}_r-\bar{\sigma}_\theta}{\bar{r}}=\bar{\rho}(\bar{r})\frac{\partial^2 \bar{u}}{\partial \bar{t}^2} \tag{10}$$

$$\bar{\sigma}_r=\bar{c}_1(\bar{r})\varepsilon_r+\bar{c}_2(\bar{r})\varepsilon_\theta \tag{11}$$

$$\bar{\sigma}_\theta=\bar{c}_1(\bar{r})\varepsilon_\theta+\bar{c}_2(\bar{r})\varepsilon_r \tag{12}$$

where

$$\varepsilon_r=\frac{\partial \bar{u}}{\partial \bar{r}},\quad \varepsilon_\theta=\frac{\bar{u}}{\bar{r}} \tag{13}$$

and

$$\bar{E}(r)=\frac{E(r)}{E_{in}}=\left(\frac{E_{out}}{E_{in}}-1\right)\left(\frac{r-r_{in}}{r_{out}-r_{in}}\right)^n+1=(\lambda-1)\left(\frac{r-r_{in}}{r_{out}-r_{in}}\right)^n+1 \tag{14}$$

$$\bar{\rho}(r)=\frac{\rho(r)}{\rho_{in}}=\left(\frac{\rho_{out}}{\rho_{in}}-1\right)\left(\frac{r-r_{in}}{r_{out}-r_{in}}\right)^n+1=(\phi-1)\left(\frac{r-r_{in}}{r_{out}-r_{in}}\right)^n+1 \tag{15}$$

The ratios $\lambda$ and $\phi$ are defined as follows:

$$\lambda=\frac{E_{out}}{E_{in}},\quad \phi=\frac{\rho_{out}}{\rho_{in}} \tag{16}$$

The terms $\bar{c}_1(r)$ and $\bar{c}_2(r)$ are

$$\bar{c}_1(r)=\frac{(1-\nu)}{(1+\nu)(1-2\nu)}\left\{(\lambda-1)\left(\frac{r-r_{in}}{r_{out}-r_{in}}\right)^n+1\right\} \tag{17}$$

$$\bar{c}_2(r)=\frac{\nu}{(1+\nu)(1-2\nu)}\left\{(\lambda-1)\left(\frac{r-r_{in}}{r_{out}-r_{in}}\right)^n+1\right\} \tag{18}$$

The stresses can be rewritten as follows:

$$\bar{\sigma}_r=\frac{1}{(1+\nu)(1-2\nu)}\left\{(\lambda-1)\left(\frac{\bar{r}-(r_{in}/r_{out})}{1-(r_{in}/r_{out})}\right)^n+1\right\}\left((1-\nu)\frac{\partial \bar{u}}{\partial \bar{r}}+\nu\frac{\bar{u}}{\bar{r}}\right) \tag{19}$$