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A domain renumbering algorithm for multi-domain boundary face method

Jianming Zhang*, Chenjun Lu, Yuan Li, Lei Han, Pan Wang, Guangyao Li

State Key Laboratory of Advanced Design and Manufacturing for Vehicle Body, Hunan University, Changsha 410082, China

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ABSTRACT

In this paper, a domain number optimization algorithm for the multi-domain boundary face method is proposed. The advantage of the algorithm is to make nonzero blocks of the overall assembled matrix are as close to the main diagonal as possible. This will minimize the block fill-in effect that occurs during the solution process. Consequently, the time used for LU-decomposition and the memory requirement of the matrix will be reduced significantly. In this algorithm, one or more level structures are generated by considering the freedom degrees and the connectivity of the domains. Then we renumber the domains according to the level structure of the smallest bandwidth. Four steady-state heat conduction problems of multi-domain are solved to test the algorithm, and high efficiency is observed.

1. Introduction

The boundary element method (BEM) is an efficient numerical technique for solving engineering problems, such as Laplace equation, Navier's equation, Helmholtz equation and linear diffusion-reaction reaction [1–6]. In the BEM, partial differential equations are converted to an equivalent boundary integral equation by Green's theorem and a fundamental solution. Thus, only boundary discretization can lead to an accurate result together with a high rate of convergence. This is the main advantage over the classic domain methods such as finite element method (FEM) and the finite difference method (FDM).

Based on the BEM, Zhang et al. have proposed the boundary face method (BFM) in recent years [7,8]. The BFM is implemented directly based on the boundary representation data structure (B-rep) that is used in most CAD packages for geometry modeling. Each bounding surface of geometry model is represented as parametric form by the geometric map between the parametric space and the physical space. Both boundary integration and variable approximation are performed in the parametric space. The integrand quantities are calculated directly from the faces rather than from elements, and thus no geometric error will be introduced. The BFM has been applied in analyses of various structural problems with complicated geometries [9–15]. In this paper, we employ the BFM to solve 3D potential problems of multi-domain.

Multi-domain formulations are employed when the entire domain under consideration is governed by individual differential equations in different parts and/or constructed of different materials [16–21]. Besides, in the case of a domain with complicated boundary profile or parallel computation, the domain may be decomposed for better computational efficiency. In a multi-domain solver, the original domain is divided into a finite number of sub-domains, and in each of them the full integral representation formula is applied. At the common interfaces between the adjacent sub-domains, the corresponding full matching conditions are enforced. How to satisfy the continuity and equilibrium conditions at the interfaces is one of the important aspects of implementation for a multi-domain algorithm. There are mainly two methods in the literature: the standard multi-domain method [22] and the domain decomposition method [23]. In the standard multi-domain method, the discretized equations corresponding to the sub-domains are assembled into a system of equation according to the boundary and interface conditions. While the matrices that arise in the single-domain formulation are fully populated, the multi-domain formulation leads to overall matrix equations with a sparse blocked structure. In the domain decomposition method, the interface conditions are assumed and then the sub-domain problems are solved independently. The modification of the interface condition is usually iterative using different methodologies, as the Schwarz Neumann–Neumann and Schwarz Dirichlet–Neumann methods. Repetition of the iteration process is continued until convergence. The domain decomposition method allows different types of discretization methods (e.g., BEM and FEM) to be used for a numerical solution of the individual sub-domains and coupling between them without accessing to the source codes of the methods. However, it has some relevant

*Correspondence to: College of Mechanical and Vehicle Engineering, Hunan University, Changsha 410082, China. Tel.: +86 731 88823061.
E-mail address: zhangjianm@gmail.com (J. Zhang).

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parameters to be chosen and the optimal values for these parameters are usually problem-dependent. This arbitrariness represents a disadvantage of the method.

In the present paper, we adopt the standard multi-domain method, and make full use of the resultant sparsity of the coefficient matrix in the equations during the solution process. The sparsity pattern (population of the nonzero blocks) of the matrix has a severe impact on the condition number of the matrix, and thus on the solution procedure. If all of the nonzero blocks are clustered near the main diagonal, we call the matrix is banded. Research indicates that reducing the bandwidth of the matrix will reduce both the memory requirement and the computation time. Many algorithms have been proposed for the problem of matrix bandwidth reduction, such as the Cuthil–McKee (CM) algorithm [24] and the Gibbs–Poole–Stockmeyer (GPS) algorithm [25]. For these algorithms, it is assumed that the connection of the nonzero elements of the matrix is continuous, and is feasible for FEM, only.

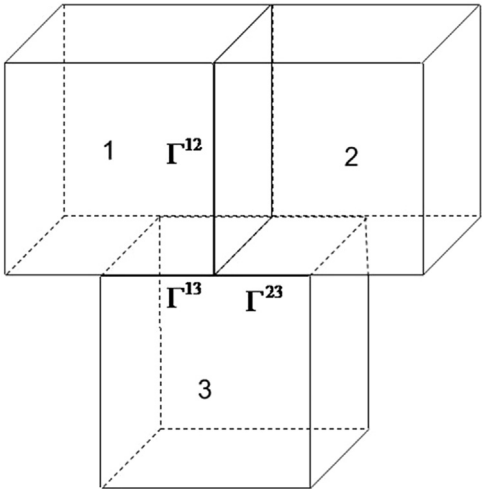

Fig. 1. Sketch map of 3 cubes.

On the implementation of the multi-domain BFM, we reduce the bandwidth of the overall assembled matrix from the following two aspects: (1) As the sparse structure of the matrix is directly related to the ordering of unknowns in the overall system of equations, we use the ordering strategy suggested by Kane [22] to obtain an optimal blocks structure. (2) A domain number optimization algorithm is proposed for the first time in this paper. The algorithm will make the nonzero blocks of the matrix as close to the main diagonal as possible. The number of the block fill-ins (to be discussed in Section 3) will be reduced correspondingly, which should make the equation solving task run much quicker. In this algorithm, firstly, one or more level structures are generated by considering the freedom degrees and the connectivity of the domains. Then the bandwidths of the level structures are

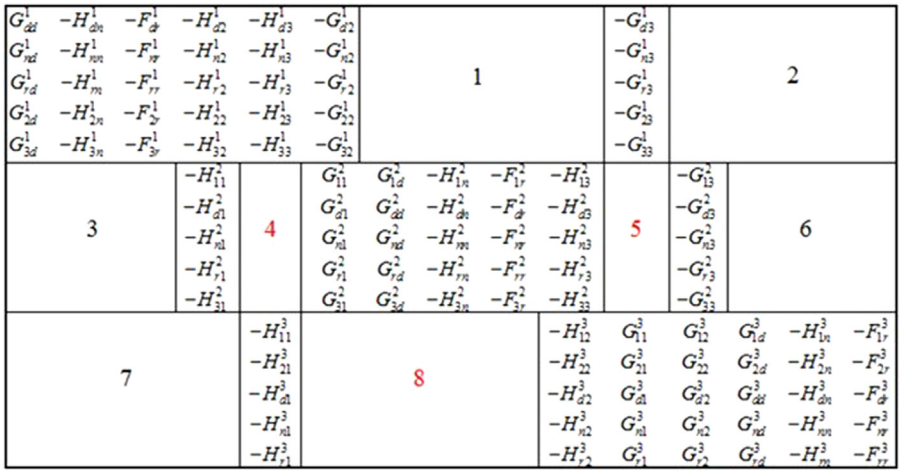

Fig. 2. Matrix with a sparse blocked structure.

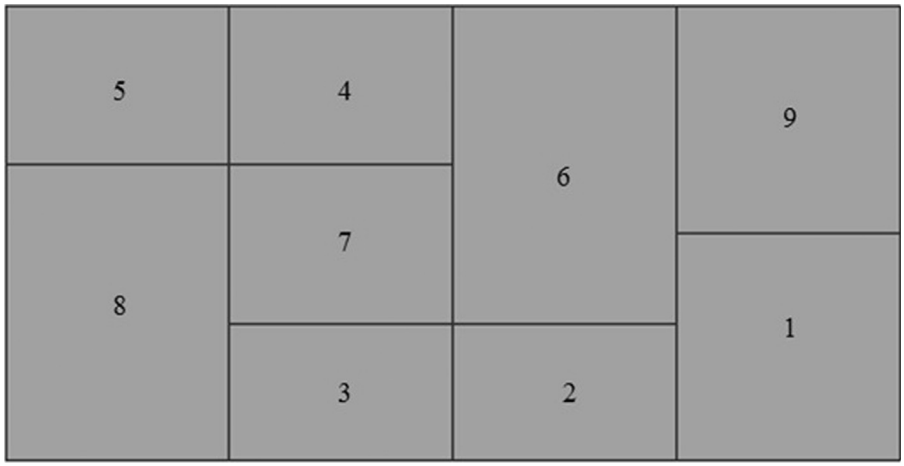

Fig. 3. Geometry and the original number the domains.