Engineering Analysis with Boundary Elements 44 (2014) 36–44

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

The complex variable reproducing kernel particle method for two-dimensional inverse heat conduction problems

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ARTICLE INFO

Article history:
Received 11 November 2013
Received in revised form
18 February 2014
Accepted 9 April 2014
Available online 8 May 2014

Keywords:
Meshless method
Reproducing kernel particle method (RKPM)
Complex variable reproducing kernel particle method (CVRKPM)
Inverse heat conduction problem

ABSTRACT

The complex variable reproducing kernel particle method (CVRKPM) for two-dimensional inverse heat conduction problems is presented in this paper. In the CVRKPM, the shape function of a two-dimensional problem is formed with one-dimensional basis function, the Galerkin weak form is employed to obtain the discretized system equation, and the penalty method is used to apply the essential boundary conditions, then the corresponding formulae of the CVRKPM for two-dimensional inverse heat conduction problems are obtained. Numerical examples are given to show that the method in this paper has higher computational accuracy and efficiency compared with the conventional element-free Galerkin (EFG) method and the reproducing kernel particle method (RKPM).

1. Introduction

The inverse problems are very important and difficult in science and engineering. Over the last several decades, the inverse heat conduction problems have gained great research interests owing to the computational challenges and practical significance. Different types of inverse heat conduction problems such as boundary inverse heat conduction problems, retrospective inverse heat conduction problems, coefficient inverse heat conduction problems and geometric inverse heat conduction problems, etc., have been extensively studied.

In general, the inverse heat conduction problems are solved by numerical methods [1–3], such as finite difference method [4], fundamental solution method [5], residual-minimization least squares method [6], wavelet method [7], finite element method [8–10], and boundary integral equation method [11–13]. These methods are derived from local interpolation schemes and require meshes to support the applications.

Meshless methods are approximate methods that are based on nodes without initial mesh and re-meshing [14–16]. Then meshless method can overcome the disadvantage that the conventional numerical method depends on the mesh of the solution domain. At present, element-free Galerkin (EFG) method [17,18], the reproducing kernel particle method (RKPM) [19–21], element-free kp-Ritz method [22–24] and the complex variable meshless method [25–29] are the most important meshless methods.

The moving least-square (MLS) approximation is employed to construct shape functions in the EFG method. The MLS approximation was developed from the conventional least-squares method, and in practical numerical processes, it essentially involved the application of the conventional method to every selected point. A new method of obtaining the approximation function named the improved MLS (IMLS) approximation has been developed by Cheng et al. [30–32]. In the IMLS, the orthogonal function system with a weight function is used as the basis function. Based on the IMLS approximation and the boundary integral equation method, boundary element-free method (BEFM) is presented for potential, elasticity, elastodynamics and fracture problems [33–37]. Based on the IMLS approximation and the Galerkin weak form, an improved EFG (IEFG) method is presented for potential, transient heat conduction, wave, elasticity, elastodynamics and fracture problems [38–44]. Using the improved interpolating MLS method to obtain the shape function, the interpolating element-free Galerkin (IEFG) method and the interpolating boundary element-free method (IBEFM) are presented for potential and elasticity problems [45–48].

The RKPM is developed on the basis of the smoothed particle hydrodynamics (SPH) method. The RKPM is one of the most important methods used to form approximation function in the

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http://dx.doi.org/10.1016/j.enganabound.2014.04.008
0955-7997/

meshless methods. The RKPM has many advantages, such as good smoothness and high computational accuracy. Based on the RKPM and Ritz method, Liew et al. presented the element-free kp-Ritz method for laminated rotating annular plate, cylindrical panel and wave Eqs. [22–24]. And Wang et al. presented Hermite reproducing kernel Galerkin meshfree method for free vibration, transient analysis and buckling analysis of thin plates [49–51].

To improve the computational accuracy and efficiency of the EFG method and the RKPM, the complex variable meshless methods were presented based on the complex theory. Based on the complex variable moving least-squares (CVMLS) approximation [52–55], the complex variable element-free Galerkin (CVEFG) method was presented for potential, elasticity, fracture, elastodynamics, transient heat conduction, advection-diffusion, viscoelasticity, elastoplasticity and large deformation problems [25–29,56–61], the complex variable boundary element-free method (CVBEFM) was presented for elastodynamics [62], and the complex variable meshless manifold method was presented for elasticity and fracture problems [63,64]. Ren et al. presented the complex variable interpolating moving least-squares method [65]. Also, introducing the complex theory into the RKPM, the complex variable reproducing kernel particle method (CVRKPM) was presented to obtain the shape function and solving transient heat conduction, elasticity, elastodynamics, elastoplasticity and advection-diffusion problems [20,66–69]. And the coupling of the CVRKPM and the finite element method was presented for solving transient heat conduction problems [70]. The CVMLS approximation and the CVRKPM are the approximations of vector functions, and the shape functions of 2D problems are formed with 1D basis function. Under the same node distribution, the meshless methods based on the CVMLS approximation and the CVRKPM have higher precision than the ones based on the MLS approximation and the RKPM, respectively. And under the similar numerical precision, the meshless methods based on the CVMLS approximation and the CVRKPM have greater computational efficiency than the ones based on the MLS approximation and the RKPM, respectively.

In this paper, the CVRKPM solving the 2D inverse heat conduction problems is proposed. For the 2D inverse heat conduction problems, the shape function of the CVRKPM is constructed, and the penalty method is employed to apply the essential boundary conditions. The Galerkin weak form is employed to obtain the discretized system equations. Then the final system equation of the 2D inverse heat conduction problems using the CVRKPM is obtained. Finite difference scheme for the time discretization of parabolic equations is used to discretize the time and to obtain the solutions of the final system equation. Some numerical examples are presented, and the CVRKPM results are compared with the ones of the RKPM and the EFG method. It is shown that the numerical results of the CVRKPM are in excellent agreement with the analytical solutions, and that the CVRKPM has higher accuracy and computational efficiency than the RKPM and the EFG method.

2. The shape function of the CVRKPM

In the CVRKPM, the approximate function $\overline{u}^h(z)$ of a complex function $\overline{u}(z)$ is [66,67]

$$\overline{u}^h(z)=\overline{u}_1^h(z)+i\overline{u}_2^h(z)=\int_\Omega \overline{u}(z')\cdot\overline{w}_h(z-z')dz',\ (z=x_1+ix_2\in\Omega), \tag{1}$$

where $\overline{w}_h(z-z')$ is a corrected reproducing kernel function,

$$\overline{w}_h(z-z')=C(z;z-z')\cdot w_h(z-z'), \tag{2}$$

$w_h(z-z')$ is the kernel function which has a compact support domain, and $C(z;z-z')$ is the correction function,

$$C(z;z-z')=\sum_{i=0}^{m}p_i(z-z')\cdot b_i(z)=\boldsymbol{p}^T(z-z')\boldsymbol{b}(z)\ ,\ (z\in\Omega), \tag{3}$$

$$\boldsymbol{p}^T(z-z')=(p_0(z-z'),p_1(z-z'),p_2(z-z'),\dots,p_m(z-z')), \tag{4}$$

$$\boldsymbol{b}^T(z)=(b_0(z),b_1(z),b_2(z),\dots,b_m(z)), \tag{5}$$

where m is the highest order of polynomial basis functions, $p_i(z-z')$ are the basis functions, and $b_i(z)$ are the corresponding coefficients.

Using the trapezoidal rule to Eq. (1), we can obtain the discretized form of the complex variables reproducing kernel approximation as

$$\overline{u}^h(z)=\boldsymbol{C}(z)\boldsymbol{W}(z)\boldsymbol{V}\cdot\overline{\boldsymbol{u}}, \tag{6}$$

where

$$\overline{\boldsymbol{u}}=(\overline{u}(z_1),\overline{u}(z_2),\dots,\overline{u}(z_n))^T=Q\boldsymbol{u}, \tag{7}$$

$$\overline{u}(z_I)=\overline{u}_1(z_I)+i\overline{u}_2(z_I), \tag{8}$$

z_I is the node in the support domain of z, n is the total number of the nodes in the support domain of z,

$$\boldsymbol{u}=(u_1(z_1),u_2(z_1),u_1(z_2),u_2(z_2),\dots,u_1(z_n),u_2(z_n))^T, \tag{9}$$

$$Q=\begin{bmatrix}1&i&0&0&0&0&\cdots&0&0\\0&0&1&i&0&0&\cdots&0&0\\0&0&0&0&1&i&\cdots&0&0\\\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&0&0&0&0&\cdots&1&i\end{bmatrix}_{n\times 2n}, \tag{10}$$

$$\boldsymbol{W}(z)=\begin{bmatrix}w(z-z_1)&0&\cdots&0\\0&w(z-z_2)&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&w(z-z_n)\end{bmatrix}, \tag{11}$$

$$\boldsymbol{V}=\begin{bmatrix}\Delta V_1&0&\cdots&0\\0&\Delta V_2&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\Delta V_n\end{bmatrix}, \tag{12}$$

ΔV_I is the volume of node z_I and represents the integration weight, i.e.

$$\sum_{I=1}^{n_t}\Delta V_I=\Omega, \tag{13}$$

and n_t is the total number of nodes distributed in the solution domain.

Let

$$C_I(z)=C_I(z;z-z_I)=\boldsymbol{p}^T(z-z_I)\boldsymbol{b}(z), \tag{14}$$

then

$$\boldsymbol{C}(z)=(C_1(z),C_2(z),\dots,C_n(z))=\boldsymbol{b}^T(z)\boldsymbol{P}, \tag{15}$$

where

$$\boldsymbol{P}=\begin{bmatrix}p_0(z-z_1)&p_0(z-z_2)&\cdots&p_0(z-z_n)\\p_1(z-z_1)&p_1(z-z_2)&\cdots&p_1(z-z_n)\\\vdots&\vdots&\ddots&\vdots\\p_m(z-z_1)&p_m(z-z_2)&\cdots&p_m(z-z_n)\end{bmatrix}. \tag{16}$$

The correction coefficients can be obtained using the reproducing conditions of the trial function, and

$$\boldsymbol{b}(z)=\boldsymbol{M}^{-1}(z)\cdot\boldsymbol{H}, \tag{17}$$