Transportation Research Part B 105 (2017) 120–143

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

An approach to transportation network analysis via transferable utility games

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A R T I C L E I N F O

Article history:
Received 19 March 2017
Revised 22 August 2017
Accepted 24 August 2017
Available online 21 September 2017

Keywords:
Network analysis
Centrality measures
Transferable utility games
Shapley value
Games on graphs

A B S T R A C T

Network connectivity is an important aspect of any transportation network, as the role of the network is to provide a society with the ability to easily travel from point to point using various modes. A basic question in network analysis concerns how "important" each node is. An important node might, for example, greatly contribute to short connections between many pairs of nodes, handle a large amount of the traffic, generate relevant information, represent a bridge between two areas, etc. In order to quantify the relative importance of nodes, one possible approach uses the concept of centrality. A limitation of classical centrality measures is the fact that they evaluate nodes based on their individual contributions to the functioning of the network. The present paper introduces a game theory approach, based on cooperative games with transferable utility. Given a transportation network, a game is defined taking into account the network topology, the weights associated with the arcs, and the demand based on an origin-destination matrix (weights associated with nodes). The network nodes represent the players in such a game. The Shapley value, which measures the relative importance of the players in transferable utility games, is used to identify the nodes that have a major role. For several network topologies, a comparison is made with well-known centrality measures. The results show that the suggested centrality measures outperform the classical ones, and provide an innovative approach for transportation network analysis.

1. Introduction

Network connectivity is an important aspect of any transportation network, as the role of the network is to provide a society with the ability to easily travel from point to point using various modes. The analysis of network connectivity can assist decision makers in identifying weak components, detecting and preventing failures, and improving connectivity in terms of decreased travel time, reduced costs, increased reliability, accessibility, etc. Various connectivity measures for general networks have been proposed, including among them the connectivity and strong connectivity of graphs (Ahuja et al., 1993); the cyclomatic number, which measures the number of circuits in a graph; the alpha index, which is the ratio between the number of existing circuits and the maximum possible number of circuits (Black, 2003; Rodrigue et al., 2006). Accessibility

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http://dx.doi.org/10.1016/j.trb.2017.08.029

measures that can be defined for transportation networks include, for example, the longest length of a shortest path in a network, which is the longest distance traveled among all shortest paths in a network. Another measure is the ratio between the lengths of the network-based shortest path and of the direct line between node pairs. Mishra et al. (2012) provide an extensive overview of connectivity measures. Some studies investigate connectivity measures related to public transportation (Hadas, 2013; Hadas and Ranjitkar, 2012).

A basic question in network analysis asks how "important" each node is. An important node might, for example, greatly contribute to short connections between many pairs of nodes, handle a large amount of traffic, generate relevant information, represent a bridge between two areas, etc. To quantify the relative importance of nodes, one possible approach uses the concept of centrality. Quite intuitively, nodes that in some sense are in the middle of a network play a major role in the functionalities of the network itself. The idea of centrality comes from the literature on social networks (Wasserman and Faust, 1994) and can be intended in different ways: in a pure topological sense, under a quasi-dynamical approach, in a dynamics-based sense, etc. Included among classical measures of centrality are degree centrality, closeness centrality, and betweenness centrality (Wasserman and Faust, 1994, chapter 10).

A limitation of classical centrality measures is that they evaluate nodes based on their individual contributions to the functioning of the network. For instance, the importance of a stop in a transportation network can be computed as the difference between the full network capacity and the capacity when the stop is closed. However, such an approach is inadequate when, for instance, multiple stops can be closed simultaneously. Consequently, the existing centrality measures need to be refined to take into account that the network nodes do not act merely as individual entities, but as members of groups of nodes.

To this end, game theory (González-Díaz et al., 2010; Tijs, 2003) provides a general basis for developing a systematic study of the relationship between rules, actions, choices, and outcomes in situations that can be either competitive or non-competitive. In an informal way, game theory is as old as social theory and was already somewhat implicitly used centuries ago by Thomas Hobbes (1588–1679), John Locke (1632–1704), and Jean-Jacques Rousseau (1712–1778) in investigating the rationale of individuals in drawing up a "social contract." The birth of the "analytical game theory" is usually identified with the publication of the book by von Neumann and Morgenstern (1944), which provides the mathematical foundations of this discipline. Afterwards, its economics-based perspective was expanded towards political science in the late 1960s and evolutionary biology in the early 1970s.

Game theory can be considered a generalization of decision theory with two or more decision makers. A game is a situation involving two or more rational decision makers called players, that make decisions in such a way that each player tries to maximize the payoff obtained as a consequence of the decisions of all players. Games can be cooperative or non-cooperative, each requiring different mathematical tools.

Game theory approaches have been used to develop transportation oriented models. For example, in a two-player non-cooperative game that was used to model the performance reliability of a network, one player tries to minimize the travel costs, while the other tries to maximize the costs (Bell, 2000). This model can assist with a cautious approach to network design, based on pessimistic users. A cooperative approach (Szeto, 2011) was developed as well, based on the Stackelberg-Nash and the partial cooperative Nash formulations to determine travel cost reliability. A non-cooperative, two-player, zero-sum game with perfect information was developed to optimize a utility function when a railway network failure occurs (Laporte et al., 2010). A game theory approach was used to model public transportation operators, various integration strategies, and market situations (Roumboutsos and Kapros, 2008). Such a model can assist the policymaker with the identification of the most cost-effective form of intervention.

The idea at the basis of game-theoretic centrality measures is that the nodes are considered players in a cooperative game, where the value of each coalition of nodes is determined by certain graph-theoretic properties. The key advantage of this approach is that nodes are ranked not only according to their individual roles in the network, but also by taking into account how they contribute to the roles of all possible groups of nodes. This is important in various applications in which the performance of a group cannot be simply described as the sum of the individual performances of each of its members. In the case of transportation networks, suppose a certain budget is available. One possible approach addresses the question of whether the investment of all the money in increasing the capacity and/or service of a transportation component (road section, bridge, transit route, bus stop, etc.) substantially improves the whole network. A preferable approach for the network designer would probably be the consideration of simultaneously improving a possibly small subset of the components. In this case, to evaluate the importance of a component one needs to take into account the potential gain of improving one among several groups of components, and not merely the potential gain of improving the component by itself. This approach can be formalized in terms of cooperative game theory (González-Díaz et al., 2010; Tijs, 2003), in which the nodes are the players and their performances are studied in coalitions that are subsets of players.

On one hand, in constructing a suitable cooperative game on a network several game-theoretic solution concepts developed during decades of research can be used. On the other hand, game-theoretic network centrality measures may be computationally very demanding. This is not an issue in the present paper that considers small networks. Moreover, recent studies (Michalak et al., 2013; Szczepański et al., 2012) show that some game-theoretic centrality measures can be efficiently computed in polynomial time with respect to the network dimension. Michalak (2016) provides a comprehensive introduction to game-theoretic network centrality.

The present study uses methods and tools from cooperative games with transferable utility, also referred to as TU games. It also uses the concept of the Shapley value (Shapley, 1953), which represents a criterion according to which each node is