Engineering Analysis with Boundary Elements 44 (2014) 185–192

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

An improved boundary distributed source method for electrical resistance tomography forward problem

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ARTICLE INFO

Article history:
Received 26 July 2013
Accepted 26 January 2014
Available online 20 February 2014

Keywords:
Boundary distributed source method
Mesh-free methods
Electrical resistance tomography
Complete electrode model

ABSTRACT

This paper presents a meshless method called the improved boundary distributed source (IBDS) method to obtain the numerical solution of an electrical resistance tomography (ERT) forward problem. The ERT forward problem contains solving the Laplace equation on piece-wise homogeneous domain subjected to the mixed boundary conditions with constraints of integral form. The IBDS method is mesh-free and does not require a fictitious boundary for source points as in the case of a conventional method of fundamental solution (MFS) approach. Therefore, it can be used for a wide variety of applications involving complex shaped objects that are difficult to mesh. Also, in the IBDS method, the diagonal elements for Neumann boundary conditions are computed analytically unlike the original BDS method. Therefore, the IBDS method is computationally efficient and stable compared to the BDS method. The ERT forward problem to compute the boundary voltages is formulated using a meshless IBDS method. Several numerical examples are tested to demonstrate the feasibility and accuracy of the new formulation. The results are compared with that of standard numerical forward solvers for ERT such as the boundary element method (BEM) and the finite element method (FEM).

1. Introduction

Electrical resistance tomography (ERT) is a non-invasive imaging technique to reconstruct the conductivity/resistivity distribution inside the domain of interest [1]. ERT image reconstruction is acquired from the relationship between injected currents and measured voltages through the electrodes that are discretely attached on the outer boundary of domain of interest [2–4]. If the conductivities of inclusion and background are assumed to be known and time invariant then the forward problem is to estimate the shape and location of inclusions [5–8]. ERT image reconstruction includes iteratively solving forward and inverse problems. The forward problem is to calculate the potential distribution for a known conductivity distribution with a certain boundary condition. The most realistic mathematical model of ERT is the complete electrode model (CEM) which takes into account the shunting effect and the contact impedance between the electrode and the substance of the domain [9,10].

Currently, the ERT forward solvers are mainly based on the finite element method (FEM) [5–8,11–15] or the boundary element method (BEM) [16–20]. Boundary estimation with the mesh based methods such as FEM in ERT is reported in [5–8,15]. However, with mesh based methods, with inclusions in the domain, the situation of mesh crossing elements can occur therefore it can lead to errors in computing the area weighted conductivities. This situation can be prevented if a very fine mesh is chosen or if it is meshed whenever the boundary of inclusion is changed such that there is no mesh crossing elements. Adaptive meshing can improve the accuracy but it results in an increase of computational burden therefore it is not a wise idea. Also, meshing is a tedious task especially if there is a complicated shape. The boundary element method that discretizes the boundaries alone is more suitable for shape estimation problems moreover it reduces the dimension by one. However, even BEM involves discretizing boundaries into line segments and boundary integrals are to be computed over the boundary therefore the computational cost is still high. In contrast, to avoid the disadvantages of mesh based methods, a comparably new class of numerical methods have been developed which approximates the partial differential equations based on a set of nodes without the need for an additional mesh, called mesh-free methods or meshless methods (MMs).

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http://dx.doi.org/10.1016/j.enganabound.2014.01.021
0955-7997

Many mesh-free (or meshless) methods have been proposed and achieved remarkable progress over the last few decades [21–37]. Among the mesh-free methods, the method of fundamental solutions (MFS) has attracted an increasing attention in many engineering and science fields [27–30]. The MFS approximates the solution of the problem as a linear combination of fundamental solutions of the governing differential operator. However, the conventional MFS approach requires a fictitious boundary outside the problem domain to place the source points due to singularity of the fundamental solution [27,28]. The determination of fictitious boundary is not trivial [28]. To overcome this main drawback of the MFS, several methods such as the modified method of fundamental solutions (MMFS) [28–31], the singular boundary method (SBM) [32–34], and the boundary distributed source (BDS) method [35–36] have been proposed. The key point of SBM is to evaluate the origin intensity factor to isolate the singularity of the fundamental solution based on subtracting and adding-back technique as well as the inverse interpolation technique. In the BDS method to avoid the singularities of the fundamental solution at source points a distributed source is considered. The distributed sources are circles in two-dimensions (2D) or spheres in three-dimensions (3D). In the BDS method, all elements of the system matrix can be derived analytically for the Dirichlet boundary conditions without singularity. For the Neumann type boundary conditions, while the off-diagonal elements are determined analytically, the diagonal elements are obtained indirectly from the constant potential field [28,35,36]. The determination of diagonal elements requires solving the system equations thus the computation time is more and also the solution is not stable. Recently, Kim [37] suggested an improved BDS (IBDS) method for the Laplace equations to determine the off-diagonal elements for the Neumann boundary conditions by using the fact that the integration of the normal derivative of the potential function over the domain boundary should vanish. In doing so, the IBDS method can remove the procedure to determine indirectly the diagonal elements for the Neumann boundary conditions.

This paper presents a meshless solution based on the IBDS method for solving the ERT forward problem. The domain is considered to be composed of more than two substances with different electrical conductivities. IBDS with a complete electrode model is derived to solve the ERT forward problem. The IBDS formulation for ERT forward problem is tested with several numerical examples for homogeneous and inhomogeneous domains. The performance of IBDS is compared against the domain and mesh based methods such as BEM and FEM. The results show a promising performance of the IBDS method as an ERT forward solver.

2. Mathematical formulation

2.1. Mathematical model for ERT

In the ERT, a set of discrete electrical currents I_l $(l=1,2,\ldots,L)$ is injected through an array of electrodes e_l $(l=1,2,\ldots,L)$ attached on the circumference of the domain $\partial\Omega$ and the excited voltages are measured on those electrodes. Assuming, an inclusion of conductivity σ_a occupying region D with a boundary ∂D is enclosed within the domain Ω having background conductivity σ_b, as shown in Fig. 1. The conductivity distribution inside the domain Ω can be represented as

$$\sigma(p)=\sigma_b+(\sigma_a-\sigma_b)\chi_D(p)=\sigma_b+\mu\chi_D(p) \quad \text{in} \quad \Omega\in\Re^2, \tag{1}$$

where p refers to the spatial location inside the domain Ω and $\chi_D(p)$ is a characteristic function of D, such that its value is 1 on D and 0 otherwise. If the conductivity distribution σ in Ω is known, then

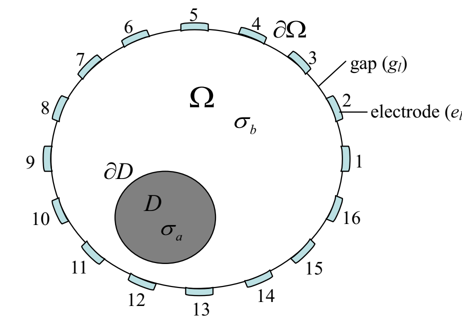

Fig. 1. A schematic diagram of ERT with 16 electrodes placed on the outer boundary.

the corresponding electrical potential u on the domain Ω can be determined from the partial differential equation, which is derived from the Maxwell equations, given by

$$\nabla\cdot\sigma\nabla u(p)=0 \quad \text{in} \quad \Omega \tag{2}$$

subjected to the following boundary conditions based on the complete electrode model (CEM) [9]:

$$\sigma_b\frac{\partial u_b}{\partial \nu}=0 \quad \text{on} \quad \partial\Omega\setminus\cup_{\ell=1}^{L}e_\ell, \tag{3}$$

$$\int_{e_l}\sigma_b\frac{\partial u_b}{\partial \nu}dS=I_l \quad \text{on} \quad e_l \ (l=1,2,\ldots,L), \tag{4}$$

$$u_b+z_l\sigma_b\frac{\partial u_b}{\partial \nu}=U_l \quad \text{on} \quad e_l \ (l=1,2,\ldots,L). \tag{5}$$

In the above equations, z_l is the effective contact impedance between the lth electrode and the electrolyte, U_l is the boundary voltage measured on the lth electrode, and ν is the outward unit normal. The interfacial boundary condition on the inclusion boundary ∂D is given by the following equation:

$$\sigma_b\frac{\partial u_b}{\partial \nu}=\sigma_a\frac{\partial u_a}{\partial \nu} \quad \text{and} \quad u_b=u_a \quad \text{on} \quad \partial D, \tag{6}$$

where u_b and u_a are the voltage potentials on the background $\Omega\setminus\overline{D}$ and the inclusion D, respectively. The outer boundary $\partial\Omega$ consists of electrode boundaries and gap boundaries, i.e. $\partial\Omega=\partial\Omega_E+\partial\Omega_G$ where $\partial\Omega_E=\cup_{l=1}^{L}e_l$ and $\partial\Omega_G=\partial\Omega\setminus\cup_{l=1}^{L}e_l=\cup_{l=1}^{L}g_l$ (Fig. 1). Also, we need the following constraints to ensure the existence and the uniqueness of the solution [10]:

$$\sum_{l=1}^{L}I_l=0 \quad \text{and} \quad \sum_{l=1}^{L}U_l=0. \tag{7}$$

2.2. Formulation of the improved boundary distributed source (IBDS) method

In the BDS method [35–36], a number of source points p_j $(j=1,2,\ldots,N)$ are selected along the domain boundary. The solution $u(p)$ at a certain field point p is expressed as a linear combination of the fundamental solution integrated over a circle $A(p_j)$ (as shown in Fig. 2) with a radius of R_j and centered at the selected source point p_j, i.e.

$$u(p)=\sum_{j=1}^{N}\int_{A(p_j)}G(p,s)\,dA\,(s)\mu_j=\sum_{j=1}^{N}\tilde{G}(p,p_j)\mu_j, \quad p\in\overline{\Omega} \quad \text{and} \quad p_j\in\partial\Omega \tag{8}$$

where μ_j are the unknown source densities to be determined, $\overline{\Omega}$ is the closure of the domain Ω, G is the fundamental solution of Laplace equation, and $\tilde{G}$ is the integration of fundamental solution G over circular disk. The fundamental solution in 2D for the