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Discrete-time day-to-day dynamic congestion pricing scheme considering multiple equilibria

Linghui Han[a], David Z.W. Wang[a,*], Hong K. Lo[b], Chengjuan Zhu[a], Xingju Cai[c]

[a] School of Civil & Environmental Engineering, Nanyang Technological University, 50 Nanyang Avenue, 639798, Singapore
[b] Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, China
[c] School of Mathematical Sciences, Jiangsu Key Labratory for NSLSCS, Nanjing Normal University, Nanjing 210023, China

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ABSTRACT

In this study, we focus on the discrete-time day-to-day dynamic congestion pricing scheme which varies the toll on a day-to-day basis and aims to drive the traffic system to a given objective traffic equilibrium state. As is well known, due to the asymmetric nature of the travel cost functions, multiple equilibria exist. In this case, without external force, the traffic system cannot converge to the traffic equilibrium state as desired by traffic management through a day-to-day adjustment process if the initial traffic state does not fall into its attraction domain (Bie and Lo, 2010). Therefore, it is imperative for traffic management to propose a traffic control measure to ensure the desired traffic state can be achieved regardless of the initial traffic state. Previous studies on the day-to-day dynamic congestion pricing, either worked on continues-time day-to-day pricing scheme, or took the form of discrete-time day-to-day pricing scheme but did not guarantee the convergence to the desired objective traffic state for the cases when multiple traffic equilibria exist. Both are undesirable. This study aims to develop a discrete-time day-to-day pricing scheme so as to direct the traffic evolution to reach the desired equilibrium from any initial traffic state when multiple traffic equilibria exist. Based on the very general formulation of day-to-day traffic dynamics model, we present a general formulation of such day-to-day pricing schemes and propose a method to obtain one specific road pricing scheme. Moreover, we present rigorous proofs and numerical tests to verify the proposed pricing scheme.

1. Introduction

Traffic congestion is one of the most challenging problems faced by many major cities, which has induced not only economic loss, but environmental pollution. It is believed that many of these costs can be prevented in principle, as they result from socially inefficient choices by individual drivers. To this end, economists have been advocating congestion pricing as an efficient way to reduce traffic congestion (Pigou, 1924; Knight, 1924; Yang and Huang, 2005). The traditional road pricing schemes are developed based on the static definition of traffic equilibrium, and the congestion pricing scheme is determined with the assumption that the desired traffic equilibrium state can eventually be achieved in a long run. However, as was

* Corresponding author.
E-mail addresses: lhhan@ntu.edu.sg (L. Han), wangzhiwei@ntu.edu.sg (D.Z.W. Wang), cehklo@ust.hk (H.K. Lo), Cjzhu@ntu.edu.sg (C. Zhu), caixingju@njnu.edu.cn (X. Cai).

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demonstrated in Horowitz (1984) and Bie and Lo (2010), for a traffic system whose equilibrium solution was known to exist, depending on the dynamic route adjustment process, the system might still fail to converge to equilibrium. Therefore it is doubtful whether the traffic system can reach the desired traffic equilibrium state or not, from any feasible initial traffic state under the pricing scheme based on static equilibrium, especially when multiple traffic equilibrium states exist.

Having noticed the problem of the pricing scheme based on static equilibrium, researchers proposed dynamic road pricing scheme. In contrast to the fixed static pricing scheme, day-to-day dynamic pricing varies the toll on a day-to-day basis. Indeed, the advent of new technologies for the travel information collection and dissemination makes it more practically feasible to implement dynamic congestion pricing. However, as compared with the congestion pricing based on static equilibrium, the research works on day-to-day dynamic congestion pricing which varies the toll on a day-to-day basis are much less. Besides, most of existing day-to-day dynamic congestion pricing scheme are developed based on continuous time day-to-day traffic dynamic model, which is conventionally expressed by differential equations. Sandholm (2002) applied evolutionary game approach to study the day-to-day dynamic congestion pricing which can force traffic system to reach system optimal state. In their study, the dynamic congestion pricing can be viewed as generalizations the marginal travel cost, and the elastic and inelastic traffic demand are both considered. Friesz et al. (2004) proposed a disequilibrium day-to-day dynamic pricing scheme, which maximizes the net social benefit over the planning period, considering drivers' day-to-day behavior articulated in continuous time and taking the form of ordinary differential equations. Fang and Szeto (2006) developed a day-to-day dynamic congestion pricing strategies that can force the traffic system to evolve from the status quo to SO. Further, Yang et al. (2007) extended their work and proposed a steepest descent day-to-day dynamic toll which can force the traffic system to evolve from the status quo to SO, and minimizes the derivative of the total system cost with regard to day t or reduces the total system cost the most for each day. Tan et al. (2015) investigated day-to-day dynamic congestion pricing schemes to minimize the system cost and time, measured in monetary and time units, respectively, with the travelers' value-of-time heterogeneity. There are also some researches on the adaptive signal control based on traffic dynamic models. For more details, one can refer to Xiao and Lo (2014), Smith (2015) and Liu and Smith (2015). This study only focuses on the dynamic road pricing schemes.

Although continuous time day-to-day traffic dynamic models possess good mathematical properties in traffic evolution, Watling and Hazelton (2003) pointed out two major defects of continuous day-to-day approaches, and recommended that discrete-time versions of day-to-day traffic dynamic models should be more suitable to describe travelers' routing choice adjustment behavior, which is assumed to be repeated daily, in accordance with daily changes in traffic flows. In addition, continuous time dynamic congestion pricing is not appropriate for real application. In the existing literature, Guo et al. (2015a) is the first to develop discrete-time dynamic congestion pricing scheme. In their work, the target traffic state is only a restraint flow state bounded by a predetermined set of traffic flow, rather than a specific traffic flow state. Although the proposed dynamic pricing scheme of Guo et al. (2015a) can direct the traffic system to converge to the restraint stable traffic flow state, the convergence proof is based on the assumption of the uniqueness of the traffic equilibrium. However, the uniqueness assumption is often violated: for example, the asymmetric travel cost function (Watling, 1996), may induce multiple traffic equilibrium states. In this case, if the initial traffic state does not fall into the attraction domain of the given objective traffic equilibrium state, without external force, traffic system cannot reach the given objective traffic equilibrium state through a day-to-day routing adjustment process. Han et al. (2016) developed specific discrete-time day-to-day congestion pricing schemes for certain existing specific traffic dynamic models. By their dynamic pricing scheme, the traffic system can converge to the desired traffic state in finite "days". However, Han et al. (2016) did not present a general formulation of dynamic pricing scheme which is applicable to the general day-to-day traffic dynamic models such as the formulation proposed by Guo et al. (2015b). This study aims to fill in the research gap by proposing a general discrete-time day-to-day dynamic congestion pricing scheme to explicitly consider the case when multiple traffic equilibria exist, which can drive the traffic system to reach a desired traffic equilibrium state through a route adjustment process as described by a general day-to-day traffic dynamic models regardless of the initial traffic state.

Specifically, in this study, we apply the very general day-to-day traffic dynamic model proposed by Guo et al. (2015b) to describe the dynamic route adjustment process. Indeed, this general day-to-day traffic dynamic model includes many existing models as particular cases. Then, we develop a general discrete-time day-to-day dynamic congestion pricing scheme to ensure that, from any feasible initial traffic state, traffic system can be directed to converge to the desired traffic state. Rigorous mathematical proof has been given to verify the model properties. Then, a specific dynamic road pricing scheme is proposed, and numerical tests are conducted to validate the pricing scheme.

This paper is organized as follows: the notations, the applied day-to-day traffic dynamic model, the problems of existing road pricing schemes are given in Section 2. In Section 3, we present a general formulation of dynamic road pricing scheme to achieve the aim of this study as well as the proof of convergence of the dynamic road pricing scheme. In Section 4, we demonstrate how to derive a specific dynamic road pricing scheme. The numerical tests are showed in Section 5. Finally, Section 6 presents the conclusions of this study and our future work.