Transportation Research Part B 104 (2017) 256–271

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

On the stochastic fundamental diagram for freeway traffic: Model development, analytical properties, validation, and extensive applications

Xiaobo Qu[a,*], Jin Zhang[b], Shuaian Wang[c,*]

[a] School of Civil and Environmental Engineering, University of Technology Sydney, Sydney, NSW 2007, Australia
[b] Griffith School of Engineering, Griffith University, Gold Coast 4222 NSW, Australia
[c] Department of Logistics & Maritime Studies, The Hong Kong Polytechnic University, Hung Hom, Hong Kong

ARTICLE INFO

Article history:
Received 14 September 2016
Revised 10 July 2017
Accepted 11 July 2017
Available online 26 July 2017

Keywords:
Stochastic fundamental diagram
Speed distributions
Traffic control

ABSTRACT

In this research, we apply a new calibration approach to generate stochastic traffic flow fundamental diagrams. We first prove that the percentile based fundamental diagrams are obtainable based on the proposed model. We further prove the proposed model has continuity, differentiability and convexity properties so that it can be easily solved by Gauss–Newton method. By selecting different percentile values from 0 to 1, the speed distributions at any given densities can be derived. The model has been validated based on the GA400 data and the calibrated speed distributions perfectly fit the speed-density data. This proposed methodology has wide applications. First, new approaches can be proposed to evaluate the performance of calibrated fundamental diagrams by taking into account not only the residual but also ability to reflect the stochasticity of samples. Secondly, stochastic fundamental diagrams can be used to develop and evaluate traffic control strategies. In particular, the proposed stochastic fundamental diagram is applicable to model and optimize the connected and automated vehicles at the macroscopic level with an objective to reduce the stochasticity of traffic flow. Last but not the least, this proposed methodology can be applied to generate the stochastic models for most regression models with scattered samples.

1. Introduction

The traffic flow fundamental diagram has been considered as the foundation of traffic flow theory. It addresses the relationship among three fundamental parameters of traffic flow: traffic flow (vehs/h), speed (km/h), and traffic density (vehs/km). As flow is the product of speed and density, this relationship usually refers to flow – density or speed – density relationship. Since the seminal Greenshields model (Greenshields et al., 1935) was proposed, numerous studies have been done to improve this over-simplified relationship empirically and/or analytically (Greenberg, 1959; Newell, 1961; Underwood, 1961; Edie, 1961; Kerner and Konhäuser, 1994; Del Castillo and Benítez, 1995a, b; Li and Zhang, 2001; Wu, 2002; MacNicolas, 2008, Ji et al., 2010; Wang et al., 2011; Wu et al., 2011; Dervisoglu, 2012; Keyvan-Ekbatani et al., 2012, 2013).

* Corresponding authors.
E-mail addresses: drxiaoboqu@gmail.com (X. Qu), wangshuaian@gmail.com (S. Wang).

http://dx.doi.org/10.1016/j.trb.2017.07.003

Table 1
Six well known speed-density models.

Models	Function	
Greenshields et al. (1935)	$v = v_f(1 - \frac{k}{k_j})$	v_f, k_j
Greenberg (1959)	$v = v_o \ln(\frac{k_j}{k})$	v_o, k_j
Underwood (1961)	$v = v_f \exp(-\frac{k}{k_o})$	v_f, k_o
Northwestern (Drake et al., 1967)	$v = v_f \exp[-\frac{1}{2}(\frac{k}{k_o})^2]$	v_f, k_o
Newell (1961)	$v = v_f\{1 - \exp[-\frac{\lambda}{v_f}(\frac{1}{k} - \frac{1}{k_j})]\}$	v_f, k_j, λ
Wang et al. (2011)	$v = \frac{v_f}{1+\exp(\frac{k-k_c}{\theta})}$	v_f, k_c, ??

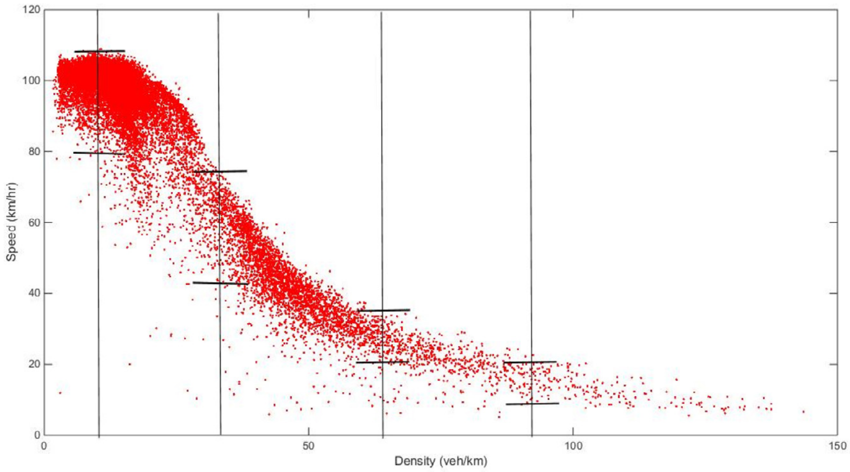

Fig. 1. GA400 speed - density sample.

The main focus of these studies is to develop accurate deterministic speed-density models with two or three practically meaningful parameters.[1]

1.1. Six prominent speed – density models

In this section, we introduce a few prominent speed – density models. Several decades after the development of Greenshields model (Greenshields et al., 1935), Greenberg (1959) propose a logarithmic function to represent this relationship. The main drawback of this model is that speed tends to infinity when density tends to zero. In order to overcome this limitation, Underwood (1961) put forward an exponential model. However, this model is not able to predict speeds at high densities. Newell (1961), Drake et al. (1967), and Wang et al. (2011) also propose their speed – density models in order to better represent this fundamental relationship. Table 1 lists six prominent speed – density models. These models can be used to determine the road capacities (Wu and Rakha, 2009), developing macroscopic traffic flow models (Phegley et al., 2013), and anticipate the traffic downstream (Kühne, 1984), and model traffic control strategies (Wang et al., 2014).

1.2. Stochasticity of speed – density samples

Although frequently being called a traffic “flow”, freeway traffic is in fact far more complex than deterministic, predictable, and homogeneous fluids governed by physical laws. Indeed, freeway traffic flow possesses inherent random characteristics as it is composed by a variety of heterogeneous vehicles with distinct mechanical and electronic features, which are driven by a group of diversified drivers with different perceptions, responses, and driving habits on freeways with varied geometric features. In fact, there is a consensus in the literature that microscopic variables of traffic flow should be modeled as random variables (e.g. Breiman, 1963; Haight, 1963; Cowan, 1971, 1975; Branston, 1976; Hoogendoorn and Bovy, 1998; Mahnke and Kaupužs, 1999; Jabari and Liu, 2012 & 2014). However, the traffic flow fundamental diagram, which refers to the relationship between speed, density, and flow, is predominantly treated as deterministic (e.g. Lighthill and Whitham, 1955; Zhang, 1998; Aw and Rascle, 2000; Zhang and Kim, 2005; Wang et al., 2011; Coifman, 2014). Fig. 1 shows the speed

[1] Note that the fundamental diagram has recently been extended to network level (i.e. macroscopic fundamental diagram), which deals with interrupted flow (e.g. Daganzo and Geroliminis, 2008; Geroliminis and Daganzo, 2008; Chiu et al., 2010; Leclercq, 2014; Keyvan-Ekbatani et al., 2015).