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Lagrangian relaxation for the reliable shortest path problem with correlated link travel times

Yuli Zhang[a], Zuo-Jun Max Shen[b,c,a,*], Shiji Song[d]

[a] Department of Industrial Engineering, Tsinghua University, Beijing 100084, China
[b] Department of Industrial Engineering & Operations Research, University of California, Berkeley, California 94720, USA
[c] Department of Civil and Environmental Engineering, University of California, Berkeley, CA 94720, USA
[d] Department of Automation, TNList, Tsinghua University, Beijing 100084, China

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ABSTRACT

Finding a reliable shortest path (RSP) in a stochastic network is a fundamental problem in transportation science. Link travel time correlation significantly affects path reliability, but also greatly increases the complexity of the RSP problem due to the quadratic form of the standard deviation term. Lagrangian relaxation (LR) based on problem reformulation, which only needs to solve a series of shortest path problems, has been recognized as an efficient method to obtain near-optimal RSPs with the optimality gap guarantee. This paper proposes a novel LR approach based on a new convex problem reformulation, and new methods to update Lagrangian multipliers and handle negative cycles of the resulting shortest path problems. Different from existing LR approaches, which adopt the classical subgradient method to solve the dual problem, a constraint generation (CG) algorithm and a subgradient projection (SP) algorithm are proposed to update Lagrangian multipliers effectively, and both algorithms are further modified to handle negative cycles. We also reveal the connection between different reformulations of the RSP problem and show that the proposed approach has a smaller duality gap than existing ones. Experiments on real transportation networks validate the effectiveness of the proposed approach in terms of convergence rate, run time, duality gap and optimality by comparison with the existing LR approaches and the outer approximation algorithm.

1. Introduction

1.1. Motivation

Travel time uncertainty is unavoidable in transportation systems due to various uncertain factors in both supply side and demand side, such as roadway capacity variation and travel demand fluctuation. In the uncertain environment, travel time reliability (TTR) has been regarded as an important criterion for travellers' routing decision. For example, Brownstone et al. (2003) have shown that travellers prefer a more reliable path rather than an unreliable one even with a smaller expected travel time.

* Corresponding author.
E-mail addresses: zhangyuli@mail.tsinghua.edu.cn (Y. Zhang), maxshen@berkeley.edu (Z.-J. Max Shen), shijis@mail.tsinghua.edu.cn (S. Song).

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To measure TTR, many criteria have been proposed, including the on-time arrival probability/percentile travel time (Frank, 1969; Nie and Wu, 2009a), effective travel time/travel time budget (Hall, 1983; Lo and Tung, 2003; Zockaie et al., 2014; Wu, 2015), α-reliable travel time (Chen and Ji, 2005; Chen et al., 2012; 2013), α-reliable mean-excess travel time (Chen and Zhou, 2009; 2010), disutility/robustness based criteria (Mirchandani and Soroush, 1987; Yin et al., 2004; Huang and Gao, 2012; Shahabi and Boyles, 2015) and others. Although these criteria are motivated by different applications, many of them have similar mathematical formulations. An important mathematical formulation of TTR takes a mean-standard deviation form: $\text{mean}(p) + \theta\sqrt{\text{variance}(p)}$ ($\theta \geq 0$), that is, the TTR of a path p is defined as a (positive) linear combination of mean and standard deviation of its travel time. In fact, the mean-standard deviation form arises in routing decision models using the effective travel time/travel time budget criterion (Lo and Tung, 2003; Zockaie et al., 2014; Wu, 2015), α-reliable travel time criterion (Chen et al., 2012; 2013), mean-excess travel time criterion (Zhang et al., 2016b), robustness criterion (Shahabi and Boyles, 2015) and others. The effectiveness of measuring TTR in terms of travel time standard deviation or variance has also been validated by Rakha et al. (2010) and Xing and Zhou (2011).

Link travel time correlation is an important factor affecting TTR since congestion on one link usually results in upstream links becoming congested, and adverse weather may cause delays on all roads in a certain area. Approaches for estimating the link travel time correlation have been developed using regression models (Gajewski and Rilett, 2004; Rachtan et al., 2013) and optimization methods (Shao et al., 2014). Through Monte Carlo simulation on real world transportation networks, Zockaie et al. (2013) show that link travel time correlation has a significant impact on TTR, and ignoring them by assuming uncorrelated link travel times would degrade the quality of resulting optimal solutions. Although the problem of finding a reliable path with uncorrelated or partially correlated link travel times can be efficiently solved by either dominance conditions based labeling/A* methods (Hutson and Shier, 2009; Nie and Wu, 2009b; Chen et al., 2012; 2013; Khani and Boyles, 2015; Chen et al., 2016a) or parametric search (Zhang et al., 2016a; 2016c), and Chen et al. (2016b) propose a deviation path algorithm to find k reliable shortest paths when link travel times are uncorrelated, the presence of link travel time correlation significantly increases its computational complexity.

This paper considers the reliable shortest path (RSP) problem with link travel time correlation of the form: $\min\{\text{mean}(p) + \theta\sqrt{\text{variance}(p)} : p \in P\}$, where $\theta \geq$ is a weight factor and P is the set of loop-less paths connecting a specific origin-destination (OD) pair. Due to the nonadditivity of its objective function, Bellman's principle of optimality does not hold for the RSP problem, even when link travel times are uncorrelated (Hutson and Shier, 2009). Therefore, we focus on devising an efficient approximation approach based on the idea of Lagrangian relaxation (LR) and analyzing its duality gap. In the remainder of this section, we first review existing algorithms for the considered problem, and then outline the proposed approach and our contributions.

1.2. Literature review

Although there are many exact solution methods for the RSP problem with uncorrelated or partially correlated link travel times, to the best of our knowledge, the outer approximation (OA) algorithm is the only one to exactly solve the RSP problem with correlated link travel times. The OA algorithm approximates the nonlinear objective function by a series of iteratively generated linear cuts, and solves the resulting master problems. Although this method demonstrates promising performance in Shahabi et al. (2013) and Shahabi and Boyles (2015) by comparison with the CPLEX solver, it suffers two deficiencies. First, its master problem is the constrained shortest path problem, which is NP-complete (Handler and Zang, 1980), and thus a commercial solver is needed, which can be a challenging task in its own right (Zockaie et al., 2014). Second, our experiments show that as the value of θ increases, that is, the weight on the nonlinear term becomes larger, the OA algorithm needs to solve a larger number of master problems with growing problem sizes.

Inexact solution methods for the RSP problem with correlated link travel times can be grouped into the following four classes.

1. *Non-dominance based method.* Under the assumption that the Cholesky decomposition matrix of the travel time covariance matrix is nonnegative, Seshadri and Srinivasan, 2012) solve the RSP problem as a multi-criteria shortest path problem and give a new permutation invariant non-dominance property. However, the assumption is restrictive and seldom holds in real-world transportation networks (Prakash and Srinivasan, 2014), and since the number of criteria is $m+1$, where m is the number of links, this method becomes computationally inefficient for large size networks.
2. *Simulation based method.* Zockaie et al. (2013) propose a simulation based method, which only needs to solve a series of shortest path problems and Zockaie et al. (2014) validate its effectiveness by comparison with the OA algorithm. However, this method requires the exact joint link travel time distribution to generate samples and does not provide the gap between its solution and the optimal one. Moreover, to guarantee its solution performance, the sample size is usually large (at least 1000 in Zockaie et al., 2014) and thus a large number of shortest path problems need to be solved. Ji et al. (2011) propose a simulation-based multi-objective genetic algorithm approach to find a set of reliable nondominant paths, which are equally good or better at least in one objective space compared to all other paths, in stochastic networks with correlated link costs.
3. *Network transformation and pruning method.* Prakash and Srinivasan (2014); 2016) propose a network transformation and pruning method to find an α-reliable shortest path. This method involves two procedures. First, dial efficient links are identified and used to construct a reduced acyclic subnetwork, and Johnson's transformation is applied to guarantee that