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Networked traffic state estimation involving mixed fixed-mobile sensor data using Hamilton-Jacobi equations

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A B S T R A C T

Nowadays, traffic management has become a challenge for urban areas, which are covering larger geographic spaces and facing the generation of different kinds of traffic data. This article presents a robust traffic estimation framework for highways modeled by a system of Lighthill Whitham Richards equations that is able to assimilate different sensor data available. We first present an equivalent formulation of the problem using a Hamilton–Jacobi equation. Then, using a semi-analytic formula, we show that the model constraints resulting from the Hamilton–Jacobi equation are linear ones. We then pose the problem of estimating the traffic density given incomplete and inaccurate traffic data as a Mixed Integer Program. We then extend the density estimation framework to highway networks with any available data constraint and modeling junctions. Finally, we present a travel estimation application for a small network using real traffic measurements obtained obtained during *Mobile Century* traffic experiment, and comparing the results with ground truth data.

1. Introduction

Transportation research is currently at a tipping point; the emergence of new transformative technologies and systems, such as vehicle connectivity, automation, shared-mobility, and advanced sensing is rapidly changing the individual mobility and accessibility. This will fundamentally transform how transportation planning and operations should be conducted to enable smart and connected communities. The transport systems can be highly beneficiated and become safer, more efficient and reliable. Nowadays, dynamic routing and traffic-dependent navigation services are available for users. Such applications need to estimate the present traffic situation and that of the near future at a forecasting horizon based on data that are available in real-time. Traffic state estimation for a road network refers to estimate all the traffic variables (e.g. cars density, speed) of the network at an instant of time based of traffic measurements. This is, for a limited amount of traffic data the estimator obtains a complete view of the traffic scenario. This estimation requires the fusion or traffic data and traffic models, the latter are typically formulated as partial differential equations (PDEs). For this framework, we will use the *Lighthill–Whitham–Richards* (LWR) partial differential equation (Lighthill and Whitham, 1956; Richards, 1956) which is commonly used to model highway traffic; derivating the model constraints is a complex problem. Other estimation tech-

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niques such as Extended Kalman Filtering (Alvarez-Icaza et al., 2004) (EKF), Ensemble Kalman Filtering (Work et al., 2010) or Particle Filtering (PF) rely on approximations to determine the model constraints, either through linearization or sampling.

Recent estimation traffic network techniques involve the Link Transmission Model (LTM) (Yperman et al., 2005; Jin, 2015) in which the Lax-Hopf formula is used to compute the demand and supply of the links and invariant junction models are used to calculate the boundary flows. Traffic estimation on a freeway segment using heterogeneous data set has also been addressed on Deng et al. (2013), is based on the establishment of a cumulative vehicle count-based state estimation models for using AVI and GPS data. Along this line, in Nantes et al. (2016) the fusion of three heterogeneous data sources on a single EKF-based estimator was proposed,they provided a comprehensive analysis of the estimation accuracy using data from loop detectors, GPS and Bluetooth scanners.

No approximation of the model is required by the framework presented on this article. An example of the usage of this framework is determining the ranges input flows (or any convex function of the boundary data) compatible with the traffic model and measurement data. The exact estimation technique presented in this paper is based on the *Moskowitz* function (Moskowitz, 1965; Newell, 1993); it is used here as an intermediate computational abstraction. The *Moskowitz* function can be understood as the integral form of the density function, and solves an *Hamilton–Jacobi* (HJ) PDE, whereas the density function itself solves the LWR PDE. An advantage of using the HJ PDE is that its solutions can be expressed semi analytically (Claudel and Bayen, 2010), which enables the derivation of the model constraints explicitly. We will now summarize the key differences between our estimation approach and the works presented earlier on this section:

- Kalman filtering considers Gaussian model noise, which is not considered in this approach. It is also considering a Gaussian error model (probabilistic), whereas the present algorithm considers a deterministic error model (for example L1, L2 or L infinity error under some threshold). Note that the present algorithm can also adjust the confidence we put on the data, which can be a term in the objective function
- In the context of the triangular diagram, the Extended Kalman filter requires mode identification (as in Munoz et al., 2003). Other approaches can be used, such as the EnKF, which do not require mode identification (Piccoli and Bayen, 2010).
- Virtually all approaches are based on the CTM (the discretized LWR model), which introduces discretization errors, unlike the approach used in the present article
- Incorporating travel time constraints requires the integration of velocities over multiple time steps, which can slow down Kalman Filter approaches significantly. In addition, the integration errors cause an additional amount of uncontrolled errors when incorporating the travel time constraints, unlike the proposed approach.
- The present approach is not a minimum mean square error estimator: the objective of the optimization problem can be chosen freely, allowing different types of problem to be solved, for instance to solve L1 norm minimization problems, to find traffic state estimates that are as sparse as possible.

1.1. Contributions of the article

The present article builds on Canepa and Claudel (2012), Canepa et al. (2013), Li et al. (2014a), Li et al. (2014b) and Anderson et al. (2013) which introduced a Mixed Integer Linear Programming framework for solving data assimilation and data reconciliation problems, for specific objective functions. In the present article, the framework initially described in Canepa and Claudel (2012) is extended to network traffic density estimation. The present article has the following contributions over the previous work presented in Canepa and Claudel (2012):

- The integration of internal traffic density data, or arbitrary travel time data (not necessarily defined as the travel time required to cross the entire physical domain), which was not considered in earlier articles (Canepa and Claudel, 2012; Canepa et al., 2013).
- The extension of the traffic state estimation framework defined in Canepa and Claudel (2012) and Canepa et al. (2013) to transportation networks, which require the proper modeling of junctions, and the integration of the entropy condition to junction flows.
- The formulation of estimation problems that do not involve a minimum variance estimation, unlike classical estimation schemes derived from the Kalman Filter. Examples of non minimum variance estimation include compressed sensing (L_1 norm minimization), shown in Section 6.

The outline of this article is the following. In Section 2 we define the solution to the LWR PDE and its equivalent formulation as a HJ PDE. In Section 3, we recall the analytical expressions of the solutions to HJ PDEs for the triangular flux functions investigated in this article, and show that the LWR PDE constraints correspond to convex constraints in the unknown initial, boundary and internal condition parameters. A first estimation example is shown in Section 4, using boundary and internal conditions from measurement data the unknown initial conditions are estimated. The framework is extended to Highway Networks in Section 6, where we also validated it using experimental traffic flow data (e.g. density, point velocity and travel time) collected during the *Mobile Century* traffic experiment.