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A BEM formulation based on Reissner's hypothesis for analysing the coupled stretching–bending problem of building floor structures

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ABSTRACT

In this work, a plate bending formulation of the boundary element method (BEM) based on the Reissner's hypothesis to perform linear analysis of plates reinforced by rectangular beams is extended to consider the beams not displayed over their middle surface. Therefore eccentricity effects are taken into account. The building floor structure is modelled as a stiffened plate which is treated as a single body without dividing it into beam and plate elements. Moreover the equilibrium and compatibility conditions are automatically imposed by the integral equations. In the proposed model the final system of equation is obtained by coupling the bending problem to the stretching problem. Besides, in order to reduce the number of degrees of freedom, both the displacements and tractions are approximated along the beam width, leading to a model where the values are defined on the beams axis. In order to validate the proposed formulation, the numerical results are compared to a well know finite element code.

1. Introduction

The boundary element method (BEM) has already proved to be a suitable numerical tool to deal with plate bending problems. The method is particularly recommended to evaluate internal force concentrations due to loads distributed over small regions that very often appear in practical problems. Moreover, the same order of errors is expected when computing deflections, slopes, moments and shear forces. Shear forces, for instance, are much better evaluated when compared with other numerical methods. They are not obtained by differentiating approximation function as for other numerical techniques.

The direct BEM formulation applied to Kirchhoff's plates has appeared in the seventies [1–3]. Soon later, in 1982, Weeën [4,5] presented a BEM formulation based on Reissner's theory to perform plate bending analysis. In [6] Katsikadelis and Yotis developed a BEM formulation for Reissner's model written in terms of a biharmonic potential and one Bessel potential as well. A connection between the classical (Kirchhoff's) and Reissner–Mindlin models is established by Palermo in [7] where the solution for bending plate analysis by considering a BEM formulation can be obtained with any of these models. It is also worth mentioning two edited books [8,9] containing BEM formulations applied to plate bending showing several important applications in the engineering context. Recently, some works have been proposed to analyse the bending problem by the BEM but considering different fundamental solutions. As example, in [10] Soares et al. developed a BEM formulation for analysing the plate bending problem by the Reissner's theory where rigid body movements have been introduced to the generalised displacement fundamental solution, resulting in modified boundary element matrices with inherent equilibrium satisfaction. In the work of Litewka and Sygulski [11] a BEM direct formulation considering the fundamental solutions of Ganowicz [12] is presented to analyse thin or moderately thick plates with various shapes, including plates with holes. Guimarães and Telles present in [13] the application of the method of fundamental solutions (MFS), a mesh-free technique, to solve cracked Reissner's plates.

Using BEM coupled with the finite element method (FEM) is the natural numerical procedure to analyse plate reinforced by beams, where the BEM is used to represent the plate elements and the FEM to approximate the beam elements. Regarding this numerical technique several formulations have already been proposed, as example the works [14–16]. However, for complex floor structures the number of degrees of freedom may increase rapidly diminishing the solution accuracy.

On the other hand there are some works where BEM is not coupled with FEM, therefore boundary elements are chosen to model both plate and beam elements. In the works published by

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Song et al. [17] and Hartmann and Zotemantel [18] have shown interesting BEM approaches to deal with building frame floors, where displacement restrictions at internal points and the use of hermitian interpolations are discussed in details. In [19–21] BEM formulations for analysing the bending problem of beam-stiffened elastic plates are proposed. Paiva and Aliabadi present in [22] a BEM formulation to analyse building floors structures which are modelled by a zoned plate with different thicknesses. In [23] the same authors present a formulation for zoned plates where the boundary integral equations of curvatures of points located at the zone's interfaces are deduced in a very easy way allowing getting the bending moments at these points easily. A BEM formulation for building floor structures in which the eccentricity effects are considered and the warping influence arising from both shear forces and twisting moments is taken into account is presented by Sapountzakis and Mokos in [24]. In [25,26] Venturini and Waidemam develop BEM formulations for elastoplastic analysis of reinforced plates and in [27] the same authors extend the previous formulation for considering geometric non-linearity as well. Wutzow present in [28] a non-linear BEM formulation for analysing reinforced porous materials, where the beam elements are modelled by the Reissner's theory applied to shell elements.

An alternative scheme to reduce the number of degrees of freedom has been proposed by Fernandes and Venturini [29] to perform simple bending analysis using a BEM formulation based on Kirchhoff's hypothesis. In the model proposed in [29], as well as in the BEM formulation based on Reissner's theory developed in [30] by Fernandes and Konda, the building floor is modelled by a zoned plate where each sub-region defines a beam or a slab. However, in [29] the bending tractions are eliminated along the interfaces, differently of the formulation proposed in [30], where those values cannot be eliminated. In order to reduce the degrees of freedom, some approximations for both displacements and tractions are adopted in [30] along the beam width, while in [29] only approximations for displacements are required since there are no tractions along interfaces. Those composed structure proposed in [29,30] are treated as a single body, being the equilibrium and compatibility conditions automatically taken into account. Then to consider the case of all beams and slabs being displayed over a same reference surface, in [31] the formulation developed in [29] is extended to incorporate the membrane effects. As the in-plane tractions are not eliminated on the interfaces, in [31] besides the bending displacements, the in-plane tractions and displacements have also to be approximated along the beam cross section to define the values on the beam skeleton line instead of interfaces.

In this work the BEM formulation considering the Reissner's theory and developed in [30], for simple bending analysis of building floor structures, is extended in order to consider the membrane effects, so that all sub-regions can be displayed over a same reference surface. Note that in the classical theory (Kirchhoff's) [32], which is appropriated for thin plates, are defined with four boundary values, besides the corner reactions: the bending moment M_n, the effective shear force V_n, the deflection w and its derivative $w_{,n}$, n being the plate boundary normal direction. The inaccuracy of the classical theory turns out to be important for thick plates, especially in the edge zone of the plate and around holes whose diameter is not larger than the plate thickness. The Reissner's theory [33], which can be used either for thin or thick plates, takes into account the shear deformation effect, being defined with six boundary values: the moments M_n and M_{ns}, the shear force Q_n, the rotations ϕ_n and ϕ_s and the deflection w, s being the plate boundary tangential direction. In order to reduce the number of degrees of freedom, the tractions and displacements for both stretching and bending problems are approximated along the beam width, resulting to a model where the values are defined on the beams skeleton lines and on the plate boundary without beams. The accuracy of the proposed model is illustrated by numerical examples whose results are compared to a well known finite element code.

2. Basic equations

Without loss of generality, let us consider the three sub-region plate depicted in Fig. 1, where h_1, h_2 and h_3 are the sub-regions thicknesses. The sub-regions are referred to as Cartesian system of co-ordinates with axes x_1, x_2 and x_3 defined on a reference surface. The plate sub-domains assumed as isolated plates are denoted by: Ω_1, Ω_2 and Ω_3, with boundaries Γ_1, Γ_2 and Γ_3, respectively. The distances of the corresponding sub-region middle surface to the reference one are given by c_1, c_2 and c_3. Alternatively, when the whole solid is considered, Γ gives the total external boundary, while Γ_{jk} represents interfaces, for which the subscripts denote the adjacent sub-regions (see Fig. 1).

Let us consider initially, the bending problem. For a point placed at any of those plate sub-regions, the plate equilibrium equations in terms of internal forces are given by:

$$M_{ij,j} - Q_i = 0 \quad i,j = 1,2 \tag{1}$$

$$Q_{i,i} + g = 0 \tag{2}$$

where g is the distributed load acting on the plate middle surface, m_{ij} are bending and twisting moments and Q_i represents shear forces.

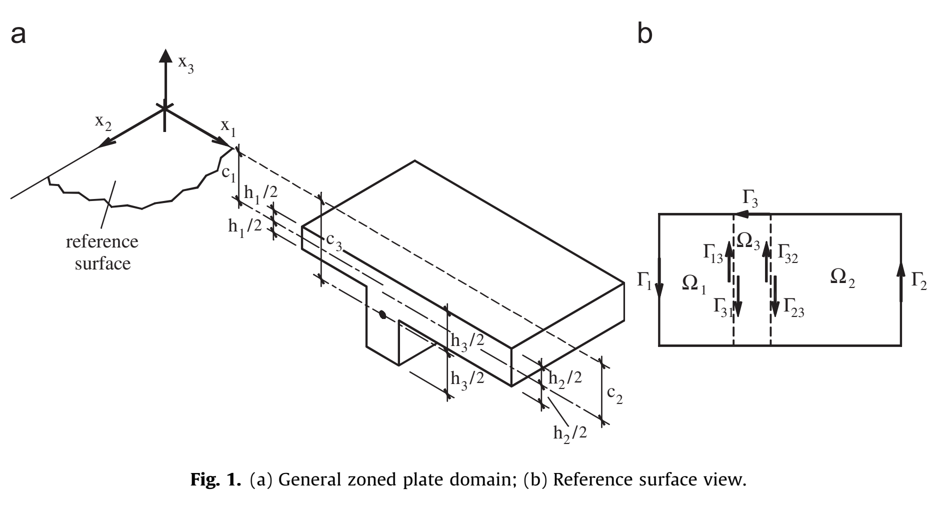

Fig. 1. (a) General zoned plate domain; (b) Reference surface view.