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# Morning commute in a single-entry traffic corridor with early and late arrivals

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ABSTRACT

The purpose of this paper is to extend the work of DePalma and Arnott (2012) by investigating the solutions for social optimum (SO) and user equilibrium (UE) assignment in a single-entry traffic corridor with consideration of both early and late arrivals. The LWR model and the Greenshields' relation are used to describe the dynamic properties of traffic flow. The closed-form SO solution and quasi-analytic UE solution are developed and well illustrated by numerical examples. It is shown that the SO assignment is associated with a smooth cumulative outflow curve, while the UE assignment will result in recursively generated cumulative inflow and outflow curves. In UE, however, the system finally becomes a free flow state.

## 1. Introduction

To date, various models have been developed to study the dynamics of the rush-hour traffic congestion. Most of them have applied the basic bottleneck model (Vickrey (1969), as simplified in Arnott et al., (1990)), in which congestion takes the form of a queue behind a single bottleneck with fixed capacity (DePalma and Arnott, 2012). Due to this simplification, the dynamic process of congestion can be analyzed in a tractable form, regardless of the specific extensions (Hendrickson and Kocur, 1981; Yang and Huang, 1997; Gonzales and Daganzo, 2012; de Palma and Fosgerau, 2013; Xiao et al., 2013; Li et al., 2014; Xiao et al., 2015; Liu et al., 2015; Wu and Huang, 2015; Xiao et al., 2016; Amirgholy and Gonzales, 2016). In these "point bottleneck" works, however, the dynamic properties of traffic stream and traffic jam were not explicitly investigated. Thus, the bottleneck model does not appear well suited to examining the spatial dynamics of traffic congestion (Arnott and DePalma, 2011).

Researchers have been trying to analyze the "flow congestion", through considering more restricted conditions on the propagation of dynamic waves along a highway or a corridor. Mahmassani and Herman (1984) stated that travelers must traverse some section of a highway rather than a bottleneck, thus the output flow should be related to the average density over the whole road section. Newell (1988) (hereinafter referred to as Newell) pointed out that the model by Mahmassani and Herman (1984) could yield an unrealistic wave velocity. In order to conquer this limitation, Newell employed the LWR model (Lighthill and Whitham, 1955; Richards, 1956) to describe the propagation of traffic flow, in which traffic is treated as a continuous anisotropic fluid whose dynamic waves propagate with a finite velocity. In this theoretical and insightful paper, Newell investigated the solutions for social optimum (SO) and user optimum (UO) of morning commute in a single-entry and single-exit corridor, and provided detailed dynamic characteristics of traffic flow at these two states. In addition,

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a more realistic phenomenon was found that the schedule delay may still exist even if the demand rate stays less than road capacity. However, this work has been ignored by the literature for a while and, conversely, much more spatial extensions of the basic bottleneck model have been developed (e.g., Kuwahara, 1990; Arnott et al., 1993; Akamatsu et al., 2015; Knockaert et al., 2016). In these papers, unlike Newell, they adopted the simplifying assumption of infinite wave velocity. Others, like Lago and Daganzo (2007) and Vincent et al., (2016), studied a more complex situation by taking the fixed wave velocity into account. Even so, the description of wave velocity and the spatial dynamics of traffic congestion in these extensions are still not as realistic as in Newell.

In order to further explore the economic properties of morning commute, DePalma and Arnott (2012) (hereinafter referred to as DA) studied a special case of the Newell model and derived the SO and UO solutions. In this special case, all commuters who travel along a single-lane road with constant width from a common origin to a common destination have a common work start time. Under these assumptions, DA explicitly showed the differences of rush-hour traffic dynamics and economic properties associated with SO and UE between the standard bottleneck model and their flow congestion bottleneck model. The marginal schedule cost, marginal variable travel time cost and marginal variable trip cost are all concave functions of population in this model. Especially, they found that those no-propagation models (Henderson, 1977; Chu, 1992; Mun, 1999; Tian et al., 2010) would overestimate the user optimum trip cost, in comparison with that by the LWR flow congestion model. They further showed that the optimal time-varying toll will change not only the departure function but also the arrival function. Meanwhile, such tolling scheme can reduce but cannot eliminate traffic congestion in the rush hour.

Obviously, the remarkable progress by Newell and DA provides a more realistic form of depicting congestion than the bottleneck model at the cost of enduring more complexity. They gave a more proximate and accurate result on understanding the spatial dynamics of rush-hour traffic congestion. What's more, it has been recognized that the analytical method is helpful for solving the general corridor problem (Arnott and DePalma, 2011). But, it should be noted that though DA obtained the closed-form solutions for SO and the quasi-analytic solutions for UO, late arriving is not allowed in their model, as well as Newell.

Thus, we in this paper further provide a detailed analysis of the SO and UO solutions for the case allowing both early and late arriving. Actually, it is more intuitive and realistic to consider the early and late arrivals together, just like the discussion in the classical bottleneck model, since both of them are essential components of morning commuting problem. However, as we will state later in this paper, taking into account late arrivals will greatly increase the complexity under the LWR flow congestion framework. In fact, DA also came to appreciate the difficulty of dealing with the discontinuity in the departure rate (from the origin) at the boundary between early and late arrivals. Thus, our work is to follow this point of entry and further contribute improvement. We will focus our attention on the continuity and discontinuity of departure rates at the boundary between early and late arrivals. Like Newell and DA, we also use the method of characteristic lines to study the congestion propagation. However, being limited to the length of our paper, we will directly quote some results from Newell and DA with simple explanations. Hence, we strongly recommend readers to read Newell and DA before perusing our paper.

Section 2 presents the model, notations and assumptions which are mostly consistent with those used by DA. Sections 3 and 4 give the main results of this study. In Section 3, the closed-form solutions for SO are derived. In this section, we first investigate the SO solution of morning commute problem with late arrivals only, and then discuss the continuity of departure rate and finally obtain the SO solution when both early and late arriving are allowed. Section 4 presents the UE solutions, along with numerical examples. Section 5 concludes our paper.

## 2. Model

Fig. 1 presents a schematic diagram of morning commuting in a single-entry traffic corridor. Since the primary results of this paper are from DA, we adopt the same notation and terminology used there. We use the word "departure" to indicate a vehicle's departure from residence (origin) and consequent entry into the corridor, and the word "arrival" to indicate a vehicle's arrival at the work place (destination) and consequent exit from the corridor. The following settings are considered.

(1) Vehicles travel along a single-lane road with length $l$ and constant width. The road has only one entry and one exit located at $x=0$ and $x=l$, respectively.
(2) Point queue may develop at the entry point if the departure rate exceeds the road capacity. The distance between origin and entry point is ignored, so does the distance between exit point and destination.
(3) All vehicles have a common desired arrival time, i.e., the work start time, $\bar{t}$. If we normalize the time such that the first departure occurs at time $t=0$, then, $\bar{t}$ and $t_f$ (departure time of the last vehicle entering the corridor) become endogenous.

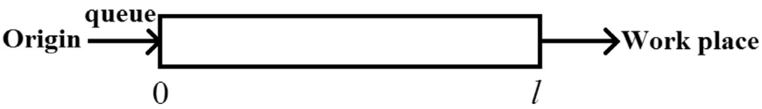

**Fig. 1.** Schematic diagram of morning commuting in a single-entry traffic corridor.