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A cooperative game approach to cost allocation in a rapid-transit network

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ABSTRACT

We consider the problem of allocating costs of a regional transit system to its users, who employ shortest path routes between all pairs of nodes in the system network. We provide an axiomatic set of conditions that a solution should satisfy and use cooperative game theory to model the cost allocation problem. We provide an allocation, called the equal cost share solution, which is efficient to compute and is the unique solution that satisfies the conditions. In addition, we show not only that the cost allocation game has a nonempty core, but further, that the game is concave, meaning that the Shapley value allocation, which coincides with the equal cost share solution, always lies in the core of the game. We provide an application of the equal cost share solution to the Washington, D.C. Metro transit network and compare it to the existing fare pricing structure. As compared to equal cost share pricing, the Metro overcharges for short downtown trips and undercharges for very long commutes. The equal cost share solution is easy to update in real time as the cost data and user distribution change, or when the transit network expands.

1. Introduction

The problem of allocating costs of rapid-transit rail systems that serve metropolitan areas is an ongoing challenge for local transportation authorities. In some transit systems, users pay a uniform fee to enter the rail system, no matter how far they travel. Other transit systems attempt to charge users in a roughly proportional fashion, by partitioning the stations into zones that radiate outward from the downtown region. Still other transit systems employ a more sophisticated fee structure where differential fares are charged for point-to-point journeys. As technology improves and smart cards become less expensive to manage, it is natural to expect the point-to-point approach to gradually become universal. In this light, the present paper proposes the use of cooperative game theory to allocate the costs of rapid-transit systems to their users in an equitable fashion.

We consider a well-defined metropolitan area in which a population of rail users makes trips on a rapid-transit system. The stations are connected to one another through a network of rail links, and we make the assumption that users who originate at a particular station will want to utilize the most efficient, or shortest, series of available links to reach their destinations. To provide these journeys will require the ability to design a shortest path network from each node in the transportation system to all other nodes. A polynomial time algorithm to determine the shortest path from one node to all others in a network was given by Dijkstra (1959). The problem of finding all shortest paths in a network was efficiently solved by Floyd (1962), using a result of Warshall (1962), and further work was done by Dantzig (1966), Moffat and Takaoka (1987), and Wang et al., (2005), among others.

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While it is easy to provide the shortest path in the transportation network for any user, our primary focus in this paper is to fairly allocate the costs of building and maintaining the rail network among the participating users. Our contribution is to provide a fair cost allocation, which is acceptable to all users and communities and can be computed efficiently, for each point-to-point journey in the transit system.

There exists an extensive literature on shortest path networks, but there is limited research on the type of cost allocation problem that we consider here. Fragnelli et al., (2000a) study shortest path games where profit is generated from transporting goods across a network, and Fragnelli et al., (2000b) study "infrastructure games" on rail systems, where the objective is to share costs among multiple transport operators, a different application from the present problem. Analogous work on sharing the costs of a linear highway was carried out by Kuipers et al., (2013). Grahn (2001) and Voorneveld and Grahn (2002) consider cooperative games on transportation networks where coalitions attempt to maximize profit derived from transport across the routes that they own, while Nebel (2010) considers the computational complexity of a model related to those of Voorneveld–Grahn and Fragnelli et al., (2000a). Rosenthal (2013) provides a Shapley value solution to a similar type of problem that we study here, but the results are restricted to tree networks.

In related transportation research, Schöbel and Schwarze (2006) examine networks where players attempt to minimize their costs, which arise from traffic on their links. Laporte et al., (2010) study railway design with possible link failures and alternate modes to offset the failed links, while Laporte et al., (2011) consider metropolitan area network planning and develop networks according to multiple objectives. Cappanera and Scappara (2011) employ game theory to allocate protective resources to a network to minimize disruptive effects like traffic delays.

In a telecommunications application, Hershberger and Suri (2001) looked at sending data along a shortest path, using Vickrey pricing to determine how much one path can save over the next best route. There has been other, related, work done in the telecommunications literature on network design and cost allocation problems (Bird, 1976; Granot and Huberman, 1981; Bergantiños and Vidal-Puga, 2010) but that literature focuses on designing least-cost networks, like spanning trees and Steiner networks, to connect users. To sum up, despite some similarities in the literature, the only prior work that addresses our dual problem of determining a fair and mutually acceptable cost allocation scheme in a shortest path network design application is Rosenthal (2013); the present paper extends that work from the special case of tree networks (that is, which do not contain cycles) to general network structures, strengthens the results, and provides an application to fare pricing in an actual rapid-transit system.

To understand the significance of this generalization, we need to briefly discuss what it would mean for some real-world examples. A cost allocation for tree networks would apply to "hub-and-spoke" transit systems, for example, in Philadelphia, Chicago (with the exception of the downtown "Loop" circuit) and Los Angeles (with one exception). Far more common, however, are transit systems that admit cycles, i.e., multiple point-to-point routes. These transit systems include Barcelona, London, New York, Paris, São Paulo, Washington D.C., and Tokyo. The preponderance of systems that are not hub-and-spoke in structure speaks to the importance of developing a satisfactory cost allocation method in the more general case.

In Section 2 we introduce conditions that a cost allocation solution for a shortest path transportation network ought to satisfy, along with the cooperative game model. In Section 3 we develop an equal cost sharing solution, which is an intuitively appealing and efficiently computable allocation. We demonstrate that the equal cost sharing solution not only coincides with the Shapley value of the cost sharing network game and is an element of the core of the game, but also satisfies the specified conditions and, further, is the unique solution to do so. In Section 4 we apply the equal cost share allocation procedure to the Washington, D.C. metro system and compare the derived passenger costs to the actual fares. In Section 5 we briefly show how the cost allocation procedure can be easily modified as the user distribution among locations changes over time, or as the network itself undergoes expansion. In addition we consider some policy implications of applying the equal cost share model, and we conclude with suggestions for further research.

2. The underlying model, solution conditions, and the cooperative game approach

2.1. The network model

To develop our model, we are given a metropolitan area M that contains a finite set $N=\{1, \ldots, n\}$ of *stations*, or *nodes*. There exists a set U of *users*, partitioned into user sets U_i, $i=1, \ldots, n$, such that $U_i \cap U_j=\emptyset$ for all $i\neq j$, and where each U_i is mapped to the station i (for $i=1, \ldots, n$) where those users start their individual origin-destination journeys. Each user takes a single trip in the transit system. For any user in set U_i, we say that station i is its *origin station* (or simply, *origin*). We represent an existing transportation network for M by an undirected graph $G=(N,E)$, where E is a set of *edges*, or *links* (s,t) between pairs of stations s and t with $s,t\in N$. We assume that there do not exist loops or parallel edges; further, we also assume that every edge in E is included in at least one origin-destination journey. We note that it is possible to generalize our model and all our results to directed networks; we discuss this in more detail in Section 5 below.

Consider user set U_i, for all $i=1, \ldots, n$. Let U_i be partitioned into sets U_{ij} for all $j\in N$ such that $j \neq i$, where each user in U_{ij} will make a single trip in G from their origin station i to destination station j. With respect to network design, we assume that each individual user $u_{ij}\in U_{ij}$ has a utility function that is based solely on his or her travel time between i and j, such that this utility strictly increases as travel time decreases. The users are assumed to be identical except for their origins, and, in order to maximize their utility, wish to travel from their origin stations i to their destinations j in the shortest possible amount of time. For any subset $U'\subseteq U$ of users, let $N^d \subseteq N$ be the set of destinations over all users in U', let $N^h \subseteq N$ be the