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A link-based mean-excess traffic equilibrium model under uncertainty

Xiangdong Xu[a], Anthony Chen[b,a,*], Lin Cheng[c], Chao Yang[a]

[a] Key Laboratory of Road and Traffic Engineering, Tongji University, Shanghai, China
[b] Department of Civil and Environmental Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong
[c] School of Transportation, Southeast University, Nanjing, China

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ABSTRACT

Traffic equilibrium models under uncertainty characterize travelers' route choice behaviors under travel time variability. In this paper, we develop a *link*-based mean-excess traffic equilibrium (L-METE) model by integrating the sub-additivity property and complete travel time variability characterization of mean-excess travel time (METT), and the computationally tractable additive route cost structure of the conventional user equilibrium (UE) problem. Compared to the majority of relevant models formulated in the *route* domain, the *link*-based modeling has two desirable features on modeling flexibility and algorithmic development. First, it avoids the normal route travel time distribution assumption (uniformly imposed for all routes) that inherits from the Central Limit Theorem in most *route*-based models, permitting the use of any suitable link travel time distributions from empirical studies. Second, the additive route cost structure makes the L-METE model solvable by readily adapting existing UE algorithms without the need of storing/enumerating routes while avoiding the computationally demanding nonadditive shortest path problem and route flow allocations in route-based models, which is a significant benefit for large-scale network applications under uncertainty.

1. Introduction

Uncertainty is present in many aspects of transportation systems. Recent empirical studies of various cities (see recent reviews by Li et al. (2010) and Carrion and Levinson (2012)) have pointed out that travel time variability plays an important role in travelers' route choice decisions. For example, Abdel-Aty et al. (1995) found that travel time reliability was either the most or second most important factor for most commuters. Travelers treat travel time variability as a risk in their travel choices, because it introduces uncertainty for an on-time arrival at the destination. On the other hand, observed travel time data exhibit a strong positive skew, very long/fat upper tail, and bimodality (see, e.g., van Lint et al., 2008; Fosgereau and Karlstrom, 2010; Susilawati et al., 2013). These empirical characteristics impose challenges to characterizing travel time distributions and modeling route choice behavior under travel time variability.

Due to the theoretical and practical importance, modeling route choice behavior and traffic equilibrium problem (TEP) under uncertainty have received great attention in the literature. Different modeling philosophies have been developed to

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[*] Corresponding author.
E-mail addresses: xiangdongxu@tongji.edu.cn (X. Xu), anthony.chen@polyu.edu.hk (A. Chen), gist@seu.edu.cn (L. Cheng), tongjiyc@tongji.edu.cn (C. Yang).

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characterize travelers' route choice behavior under travel time variability, e.g., the expected utility (Mirchandani and Soroush, 1987; Tatineni et al., 1997; Chen et al., 2002b; Yin et al., 2004; Watling, 2006), reliability (Lo et al., 2006; Shao et al., 2006; Lam et al., 2008; Nie and Wu, 2009; Chen and Zhou, 2010), prospect theory (Connors and Sumalee, 2009; Xu et al., 2011), game theory (Bell and Cassir, 2002; Szeto et al., 2006), ambiguity-aware CARA (constant absolute risk aversion) travel time model (Qi et al., 2016), stochastic dominance (Wu and Nie, 2011), and multi-objective optimization (Tan et al., 2014; Wang et al., 2014), etc.

The estimation of risk-taking route choice criteria typically requires the knowledge of route travel time probability distribution. In the literature, most TEPs under uncertainty (see some exceptions to be discussed in Section 2.1) use the Central Limit Theorem (CLT) to propagate link travel time distributions (using the mean and variance only). The CLT implicitly assumes *route* travel times to be *normally distributed* and *link* travel times to be *independent*[1], regardless of *link* travel time distributions. This assumption enables a simple approximation of route travel time distributions. However, it does not necessarily concur with empirical characteristics (i.e., strong positive skew, very long/fat upper tail, and bimodality). Also, besides normal distribution, diverse distribution types have been used to fit travel time distribution. Examples include Lognormal (Emam and Al-Deek, 2006; Kaparias et al., 2008; Rakha et al., 2010; Arezoumandi, 2011; Chen et al., 2014), Shifted Lognormal (Srinivasan et al., 2014), Weibull and Exponential (Al-Deek and Emam, 2006), Gamma (Polus, 1979), compound Gamma (Kim and Mahmassani, 2015), Generalized Beta (Castillo et al., 2012), Stable (Fosgerau and Fukuda, 2012), and Burr (Susilawati et al., 2013). Hence, the route-based CLT modeling approach may sacrifice too much modeling flexibility (i.e., realistic route travel time distribution and link travel time interdependence) for mathematical tractability.

Other than the above limitation on modeling flexibility, the *route*-based modeling approach also adds complexity to algorithmic development for applications of large-scale networks. The main reason is that the risk-taking route choice criteria, e.g., the widely used travel time budget (TTB), percentile travel time (PTT), and mean-excess travel time (METT), are typically nonadditive. In other words, route cost cannot be easily decomposed into the sum of link costs. Solving the nonadditive TEPs needs either a nonadditive shortest path algorithm (Lo and Chen, 2000; Chen et al., 2001, 2012) to serve as a column generation scheme or a priori working route set created by a route choice set generation scheme (Prato, 2009). The computation of either scheme is quite expensive for large-scale networks, which also hinders the applications of bi-level optimization problems when using the TEP models as the lower-level subprogram.

This paper considers a recent addition to the family of traffic equilibrium models under uncertainty, known as the mean-excess traffic equilibrium (METE) model (Chen and Zhou, 2010). The METE model adopts the *route*-based METT to capture both reliability (on-time arrival) and unreliability (late arrival) aspects of route travel time variability. Thus, METT can be regarded as a more complete and accurate risk-averse measure to describe travelers' route choice decisions under uncertainty. Also, it has received other applications such as in modeling travelers' stochastic perception error (Chen et al., 2011; Xu et al., 2013), demand elasticity under multiple user classes (Xu et al., 2014b), routing hazardous materials on time-dependent networks (Taumazis and Kwon, 2013), travel time robust reliability (Sun and Gao, 2012), strategies cost in schedule-based transit networks with capacity constraints (Rochau et al., 2012), risk-based transit schedule design (Zhao et al., 2013), network performance assessment (Xu et al., 2014a), and parking pricing and modal split (Zhu et al., 2014). However, the route-based METT still has the above two shortcomings on modeling flexibility and algorithmic development. In this paper, we address the possibility of formulating the TEP under uncertainty in the *link* domain and propose a *link*-based METE model to resolve the two shortcomings simultaneously.

Specifically, we develop a *link*-based mean-excess traffic equilibrium (L-METE) model by making use of the sub-additivity property of mean-excess travel time (METT). The sum of *link*-based additive METTs on a route provides an upper bound of the *route*-based nonadditive METT. Conceptually, the L-METE model integrates the sub-additivity property of METT, the complete travel time variability characterization of METT, and the computationally tractable additive route cost structure of the conventional user equilibrium (UE) problem. Similar to the classical Beckmann transformation, two equivalent mathematical programming (MP) formulations (i.e., link-route and node-link) are provided for the *link*-based METE model. Compared to the majority of relevant models formulated in the *route* domain, the modeling philosophy in the *link* domain has two desirable features.

(1) *Modeling flexibility*: the *link*-based METE model has no specific assumptions on link and route travel time distributions. Any suitable link travel time distribution from empirical studies can be adopted. Hence, it avoids the assumptions of independent (or weakly dependent) link travel times and normally distributed route travel times that inherited from the Central Limit Theorem in most *route*-based models.

(2) *Algorithmic development*: the existing algorithms for the conventional UE problem in the planning software packages can be readily adapted to solve the L-METE model while avoiding solving the nonadditive shortest path problem in route-based models. The additive route cost structure makes the L-METE model solvable without storing routes, which is a significant benefit for planners working with large-scale networks. For demonstration purposes, we customize the widely used Frank-Wolfe algorithm of the UE problem to solve the L-METE model and then apply it to a large-scale realistic network.

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[1] The CLT has a number of variants. The basic version states that given certain conditions, the mean of a sufficiently large number of independent random variables will be approximately normally distributed, regardless of the underlying distribution. Further generalizations have weakened the independence assumption and allow the random variables to be weakly dependent or not "too" dependent (Rice, 2007).