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Parsimonious shooting heuristic for trajectory design of connected automated traffic part I: Theoretical analysis with generalized time geography

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ABSTRACT

This paper studies a problem of designing trajectories of a platoon of vehicles on a highway segment with advanced connected and automated vehicle technologies. This problem is very complex because each vehicle trajectory is essentially an infinite-dimensional object and neighboring trajectories have complex interactions (e.g., car-following behavior). A parsimonious shooting heuristic algorithm is proposed to construct vehicle trajectories on a signalized highway segment that comply with boundary conditions for vehicle arrivals, vehicle mechanical limits, traffic lights and vehicle following safety. This algorithm breaks each vehicle trajectory into a few sections that are analytically solvable. This decomposes the originally hard trajectory design problem to a simple constructive heuristic. Then we slightly adapt this shooting heuristic algorithm to efficiently solve a leading vehicle problem on an uninterrupted freeway. To study theoretical properties of the proposed algorithms, the time geography theory is generalized by considering finite accelerations. With this generalized theory, it is found that under mild conditions, these algorithms can always obtain a feasible solution to the original complex trajectory design problem. Further, we discover that the shooting heuristic solution is a generalization of the solution to the classic kinematic wave theory by incorporating finite accelerations. We identify the theoretical bounds to the difference between the shooting heuristic solution and the kinematic wave solution. Numerical experiments are conducted to verify the theoretical results and to draw additional managerial insights into the potential of trajectory design in improving traffic performance. In summary, this paper provides a methodological and theoretical foundation for advanced traffic control by optimizing the trajectories of connected and automated vehicles. Building upon this foundation, an optimization framework will be presented in a following paper as Part II of this study.

1. Introduction

1.1. Background

As illustrated by the trajectories in the time-space diagram in Fig. 1(a), traffic on a signalized arterial is usually forced to decelerate and accelerate abruptly as a result of alternating green and red lights. When traffic density is relatively high,

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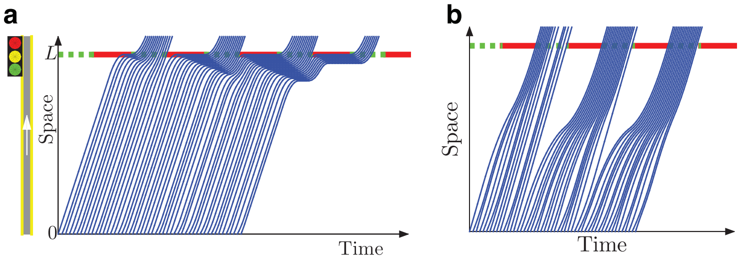

Fig. 1. Vehicle trajectories along a highway segment upstream of a signalized intersection: (a) benchmark manual vehicle trajectories; (b) smoothed automated vehicle trajectories.

stop-and-go traffic patterns will be formed and propagated backwards along shock waves. Similar stop-and-go traffic also occurs frequently on freeways even without explicit signal interruptions. Such stop-and-go traffic imposes a number of adverse impacts to highway performance. Obviously, vehicles engaged in abrupt stop-and-go movements are exposed to a high crash risk (Hoffmann and Mortimer, 1994), not to mention extra discomfort to drivers (Beard and Griffin, 2013). Also, frequent decelerations and accelerations cause excessive fuel consumption and emissions (Li et al., 2014), which pose a severe threat to the urban environment. Further, when vehicles slow down or stop, the corresponding traffic throughput decreases and the highway capacity drops (Cassidy and Bertini, 1999), which can cause excessive travel delay.

Although stop-and-go traffic has been extensively studied in the context of freeway traffic with either theoretical models (e.g., Bando et al., 1995; Herman et al., 1958; Li and Ouyang, 2011) or empirical observations (e.g., Ahn, 2005; Kerner and Rehborn, 1996; Kuhne, 1987; Laval, 2011; Li and Ouyang, 2011; Mauch and Cassidy, 2002), few studies had investigated how to smooth traffic and alleviate corresponding adverse consequences on signalized highways until the advent of vehicle-based communication (e.g., connected vehicles or CV) and control (e.g., automated vehicles or AV) technologies. CV basically enables real-time information sharing and communications among individual vehicles and infrastructure control units.[1] AV aims to replace a human driver with a robot that constantly receives environmental information via various sensor technologies (as compared to human eyes and ears), and consequently determines vehicle control decisions (e.g., acceleration and braking) with proper computer algorithms (as compared to human brains) and vehicle control mechanics (as compared to human limbs)[2] The combination of these two technologies, referred to as connected and automated vehicles (CAV), enables disaggregate control (or coordination) of individual vehicles with real-time vehicle-to-vehicle and vehicle-to-infrastructure communications. Before these technologies, highway vehicle dynamics was determined by microscopic human driving behavior. However, there was not even a universally accepted formulation of human driving behavior (Treiber et al., 2010) due to the unpredictable nature of humans (Kerner and Rehborn, 1996) and limited empirical data to comprehensively describe such behavior (Daganzo et al., 1999). Therefore, it was very challenging, if not completely impossible, to perfectly smooth vehicle trajectories with traditional infrastructure-based controls (e.g., traffic signals) that are designed to accommodate human behavior. Whereas CAV enables the replacement (at least partially) of human drivers with programmable robots whose driving algorithms can be flexibly customized and accurately executed. This opens up opportunities to control individual vehicle trajectories in coordination with aggregate infrastructure-based controls such that both individual drivers' experience and overall traffic performance can be optimized. These opportunities inspired several pioneering studies to explore how to utilize CAVs to improve mobility and safety at intersections (Dresner and Stone, 2008; Lee and Park, 2012) and reduce environmental impacts along highway segments (Ahn et al., 2013; Yang and Jin, 2014). However, these limited studies mostly focus on controlling one or very few vehicles at a particular highway facility (e.g., either an intersection or a segment) to achieve a certain specific objective (e.g., stability, safety or fuel consumption) rather than smoothing a stream of vehicles to improve the overall traffic performance. Most of the developed control algorithms require sophisticated numerical computations and their real-time applications might be hindered by excessive computational complexities.

This study aims to propose a new CAV-based traffic control framework that controls detailed trajectory shapes of a stream of vehicles on a stretch of highway combining a one-lane segment and a signalized intersection. As illustrated in Fig. 1(b), the very basic idea of this study is smoothing vehicle trajectories and clustering them to platoons that can just properly occupy the green light windows and pass the intersection at a high speed. Note that a higher passing speed indicates a smaller car-following time headway or a larger throughput, and thus we see that the CAVs in Fig. 1(b) not only have much smoother trajectories but also spend much less travel times compared with the benchmark manual vehicles in Fig. 1(a). Further, smoother trajectories imply safer traffic, less fuel consumption and emissions, and better driver experiences. While the research idea is intuitive, the technical development is quite sophisticated, because this study needs to manipulate continuous trajectories that not only individually have infinite control points but also have complex interactions between one another due to the shared right-of-way. In order to overcome these modeling challenges, we first partition each trajectory into a few parabolic sections that are analytically solvable. This essentially reduces an infinite-dimensional trajectory into a

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[1] http://www.its.dot.gov/connected_vehicle/connected_vehicle.htm.

[2] http://en.wikipedia.org/wiki/Autonomous_car .