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A Riemann solver for a system of hyperbolic conservation laws at a general road junction

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ABSTRACT

The kinematic wave model of traffic flow on a road network is a system of hyperbolic conservation laws, for which the Riemann solver is of physical, analytical, and numerical importance. In this paper, we present a new Riemann solver at a general network junction in the demand-supply space. In the Riemann solutions, traffic states on a link include the initial, stationary, and interior states, and a discrete Cell Transmission Model flux function in interior states is used as an entropy condition, which is consistent with fair merging and first-in-first-out diverging rules. After deriving the feasibility conditions for both stationary and interior states, we obtain a set of algebraic equations, and prove that the Riemann solver is well-defined, in the sense that the stationary states, the out-fluxes of upstream links, the in-fluxes of downstream links, and kinematic waves on all links can be uniquely solved. In addition, we show that the resulting global flux function in initial states is the same as the local one in interior states. Hence we presents a new definition of invariant junction models, in which the global and local flux functions are the same. We also present a simplified framework for solving the Riemann problem with invariant junction flux functions.

1. Introduction

A better understanding of traffic dynamics on a road network is critical for improving safety, mobility, and environmental impacts of modern surface transportation systems (Schrank and Lomax, 2009): practically, it is helpful for efficient implementations of ramp metering (Papageorgiou and Kotsialos, 2002), evacuation (Sheffi et al., 1982), signal control, and other management and control strategies; theoretically, it can yield better network loading models for many other transportation network studies (Wu et al., 1998). In a road network, vehicular traffic dynamics can be described by cellular automata models (Nagel and Schreckenberg, 1992; Daganzo, 2006) and car-following models (Gazis et al., 1961) of individual vehicles' movements, fluid dynamic models of continuous car-following behaviors (Payne, 1971; Whitham, 1974; Aw and Rascle, 2000; Zhang, 2002), the Lighthill–Whitham–Richards (LWR) kinematic wave model (Lighthill and Whitham, 1955; Richards, 1956), or regional continuum models (Beckmann, 1952; Ho and Wong, 2006). The traditional LWR model describes traffic dynamics of homogeneous vehicles on a virtually one-lane road as combinations of shock and rarefaction waves and can be analyzed with theories of hyperbolic conservation laws (Lax, 1972). With a balance between physical reality and math-

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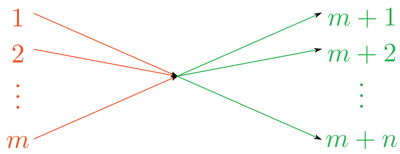

Fig. 1. A general network junction.

ematical tractability, kinematic wave models have been successfully extended to study more complicated traffic dynamics of heterogeneous vehicles (Benzoni-Gavage and Colombo, 2003), on multi-lane roads (Daganzo, 1997), and through network junctions (Holden and Risebro, 1995).

In a road network, such bottlenecks as merging, diverging, and general junctions play a critical role in initiating, propagating, and dissipating traffic congestion. Some interesting network-wide traffic phenomena can be caused by interactions among these network bottlenecks: for examples, a beltway network can be totally gridlocked (Daganzo, 1996), and periodic unstable oscillations can occur in a diverge-merge network (Jin, 2009, 2013). Thus efforts are warranted to develop both physically realistic and mathematically tractable kinematic wave models of traffic dynamics through a general network junction shown in Fig. 1, which has m upstream links and n downstream links. In the literature, there have been three lines of research toward the goal. In the line of discrete Cell Transmission Model (CTM) (Daganzo, 1995; Lebacque, 1996), boundary fluxes through a junction during a time interval are prescribed from adjacent cells' conditions based on macroscopic merging and diverging rules. In the line of conservation laws (Holden and Risebro, 1995; Garavello and Piccoli, 2006b), shock and rarefaction waves on all links are analytically solved with jump initial conditions by decoupling the Riemann problem at the junction into $m+n$ Riemann problems on individual links. In the third line by integrating the first two approaches (Jin, 2012a), the discrete flux functions in CTM are used as decoupling conditions, and it was shown that the Riemann problem can be uniquely solved. The first two approaches bear their respective limitations: the CTM approach is physically realistic and has been verified by empirical observations but cannot be applied to obtain such analytical insights as shock and rarefaction waves at a junction; the conservation law models are mathematically tractable, but the decoupling method based on an optimization problem is not directly associated with any physical merging or diverging rules. In contrast, the third approach integrates the physical merging and diverging rules in CTM into the conservation law framework has the potential to be both physically realistic and mathematically tractable.

Since the kinematic wave model of junction traffic is a system of hyperbolic conservation laws, solutions to the Riemann problem, in which all links carry constant initial conditions, but discontinuities can occur at the junction, are of physical, analytical, and numerical importance: physically, they can define macroscopic merging, diverging, and other behavioral rules; analytically, a system of hyperbolic conservation laws is well-defined if and only if the Riemann problem is uniquely solved (Bressan and Jenssen, 2000); and numerically, they can be incorporated into the Godunov finite difference equations (Godunov, 1959).

In this study, we present a new Riemann solver of the junction kinematic wave model using the solution framework of Jin (2012a), which is meant to be physically realistic and mathematically tractable. Note that the Riemann solver is analytical in the sense of Garavello (2011), different from numerical ones as discussed in Roe (1981). Mathematically, the new solver is based on that in Holden and Risebro (1995): in the Riemann solutions, a stationary state arises on a link along with a shock or rarefaction wave, which is determined by the Riemann problem of the LWR model on the link with the initial and stationary states; the stationary state should be inside a feasible domain, such that the shock or rarefaction wave propagates backward on an upstream link and forward on a downstream link; and the constant in- or out-flux of a link equals the stationary flow-rate. The remaining piece in the Riemann solver is to introduce an entropy condition such that "this condition gives a unique solution at least for Riemann initial data" (Holden and Risebro, 1995). But physically, different from that in Holden and Risebro (1995), the entropy condition for the Riemann solver in this study is a discrete flux function in terms of upstream demands and downstream supplies, which was originally developed within the framework of CTM (Daganzo, 1995; Lebacque, 1996). Thus the flux function can model conflicts among merging and diverging traffic streams at the aggregate level, and is therefore a natural choice as the entropy condition to pick out unique physical solutions. To incorporate the new entropy condition, we enlarge the function space for weak solutions to the Riemann problem by introducing on each link an interior state, which is local and takes no space (of measure zero) right next to the junction. Then the entropy condition is introduced so that boundary fluxes through the junction be determined locally from interior states with numerical CTM flux functions.

This study is closely related to but substantially different from Jin (2012a). Both studies use the same integrated framework for solving the Riemann problem; i.e., mathematical solutions are defined in the sense of Holden and Risebro (1995), but the physical entropy condition is based on the CTM junction flux function (Daganzo, 1995; Lebacque, 1996). However, in Jin (2012a), the entropy condition was the junction flux from Jin and Zhang (2004), but in this study the entropy condition is the global flux function derived in Jin (2012a). More importantly here we systematically develop a new Riemann solver and present a new definition of invariant junction models, which were first defined in Lebacque and Khoshyaran (2005). As a side product, this study also proves that the general junction model derived in Jin (2012a) is invariant.