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On the stability of stationary states in general road networks

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ABSTRACT

In [Jin, W.-L., 2015. On the existence of stationary states in general road networks. Transportation Research Part B 81, 917–929.], with a discrete map in critical demand levels, it was proved that there exist stationary states for the kinematic wave model of general road networks with constant origin demands, route choice proportions, and destination supplies. In this study we further examine the stability property of stationary states with the same map, and the results will help us to understand the long-term trend of a network traffic system. We first review a network kinematic wave model and properties of stationary states on a link, define the criticality of junctions in stationary states, and discuss information propagation in stationary states on links and junctions. We then present the map and examine information propagation in the map. We apply the map to analytically study the stability of stationary states on ring roads and diverge-merge networks with circular information propagation and compare them with results obtained from the Poincaré map [Jin, W.-L., 2013. Stability and bifurcation in network traffic flow: A Poincaré map approach. Transportation Research Part B 57, 191–208]. We further study the stability property of general stationary states in a grid network. We find that the stability of fixed points of the map is the same as that of stationary states in a network, and the new approach is more general than the Poincaré map approach. We conclude the study with future directions and implications.

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1. Introduction

One of the core tasks in road transportation research is to analyze, control, manage, plan, and design road networks during peak periods. Since "the traffic demand and origin-destination desires are relatively constant over the time period" (Wattleworth, 1967), it has been assumed that traffic patterns are relatively stationary during a peak period in many transportation network problems. Such an assumption has been underlying the static traffic assignment problem (Beckmann et al., 1956; Merchant and Nemhauser, 1978), the local and global control problem of a freeway system (Wattleworth, 1967), the existence of a network-wide macroscopic fundamental diagram (MFD) (Godfrey, 1969; Jin et al., 2013), the network flow problem (Potts and Oliver, 1972), and the integrated traffic assignment and ramp metering problem (Payne and Thompson, 1974; Yang and Yagar, 1995; Yang and Lam, 1996). Therefore it is important to understand properties of such stationary states.

In the literature, there are very few studies on the definition, existence, stability, and other properties of stationary states in road networks. In Daganzo (1996, 2007), the existence and stability properties of gridlock states were discussed for both

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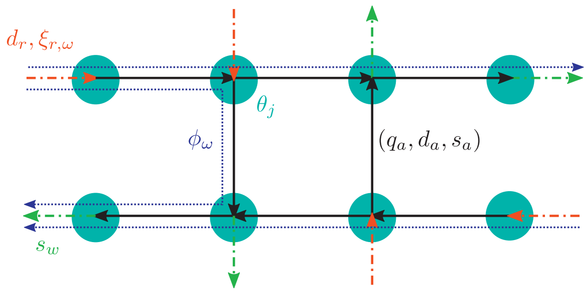

Fig. 1. A grid network. (For interpretation of the references to colour in the text, the reader is referred to the web version of this article.)

beltway and arterial networks with kinematic wave models. Such gridlock states are a special type of stationary states, since a gridlocked network will always be in the gridlock state if no mitigation strategies are implemented. In Jin (2012c), stationary states were first defined and then solved by enumeration for a diverge-merge network within the framework of a network kinematic wave model. In Jin (2013), a Poincaré map in fluxes was derived for stationary states in beltway and diverge-merge networks with circular information (wave) propagation and applied to analyze their stability property. In Jin (2015), some observations on the Los Angeles freeway network during the morning peak hours on June 18, 2013 were presented to support that the locations and sizes of queues are nearly time-independent; i.e., it is a reasonable assumption that traffic is stationary during peak periods. Further, the existence of stationary states was proved for a network kinematic wave model with constant origin demands, route choice proportions, and destination supplies; in particular, a discrete map in critical demand levels was constructed for general road networks, and the existence of its fixed point and, therefore, the corresponding stationary state was proved with Brouwer's fixed point theorem. As in other dynamical systems, the stability property of stationary states is also fundamentally important, as it determines the long-term trend of a network traffic system. However, there exists no approach to analyze the stability property of stationary states in general road networks.

In this study, we attempt to fill the gap by investigating the stability property of stationary states in general road networks with the help of the new map in critical demand levels developed in Jin (2015). One objective is to show that the stability property of the map's fixed points is the same as that of the corresponding stationary states. Another objective is to show that the new approach based on the map in critical demand levels is more general than the Poincaré map approach in Jin (2013) as it applies to road networks of any structure. To achieve these objectives, we first discuss properties of stationary states, by especially defining the criticality of junctions and discussing information propagation in the new map. Then we analytically derive the stability condition of the map's fixed points for stationary states with circular information propagation in beltway and diverge-merge networks and show the consistency in stability for the new approach and the one based on the Poincaré map in Jin (2013). With the new approach we further discuss the stability property of general stationary states in a 2×2 grid network, for which the Poincaré map no longer applies, and verify the results with the Cell Transmission Model (Daganzo, 1995).

The rest of the paper is organized as follows. In Section 2, we review the network kinematic wave model and properties of stationary states on links, define the criticality of junctions, and discuss information propagation on stationary links and junctions. In Section 3, we present the discrete map in critical demand levels and discuss information propagation in the map regarding inter-dependence of critical demand levels. In Section 4, we analytically study the stability of stationary states in beltway and diverge-merge networks with circular information propagation. In Section 5, we study the stability of stationary states in grid networks. Finally in Section 6 we conclude with some future studies.

2. A network kinematic wave model and stationary states

A general road network, e.g., a grid network shown in Fig. 1, has the following components:

- Links. The set of origin links is denoted by R (dash-dotted red lines), the set of destination links by W (dash-dotted green lines), the set of regular links by A (solid black lines), and the set of all links by $A' = R \cup W \cup A$. All the links are directed. The origin and destination links can be dummy links with zero lengths.
- Junctions. The set of junctions is denoted by J (cyan disks). For junction $j \in J$, the set of upstream (incoming) links is denoted by I_j, and the set of downstream (outgoing) links by O_j.
- Commodities. The set of commodities is denoted by Ω. In this study, vehicles using the same route (blue dashed line) belong to the same commodity; but we can further differentiate commodities according to vehicle classes and other characteristics. The set of commodities using link $a \in A'$ is denoted by Ω_a. Without loss of generality, we assume that all routes are acyclic.

For example, the network in Fig. 1 has four origin links, four destination links, eight regular links, eight junctions, and 16 routes and commodities. For link a, whose length is L_a, we denote the longitudinal coordinate by $x_a \in [0, L_a]$, which increases in the traffic direction. In addition, at x_a and time t, the total density, speed, and flow-rate are denoted by $k_a(x_a,$

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