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Extending the Link Transmission Model with non-triangular fundamental diagrams and capacity drops

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A B S T R A C T

The original Link Transmission Model as formulated by Yperman et al. (2006) simulates traffic according to Lighthill–Whitham–Richards theory with a very small numerical error, yet only supports triangular fundamental diagrams. This paper relaxes that restriction in two steps. Firstly, we extend the model to handle any continuous concave fundamental diagram, and prove that this extension is still consistent with Lighthill–Whitham–Richards theory. Secondly, we extend the theory and model to handle a capacity drop, explicitly looking into the handling of both the onset and release of congestion. The final model is still first-order and suitable for general networks. Numerical examples show that it qualitatively improves on the original model due to uniquely featuring complex traffic patterns including stop-and-go waves, with crisp shockwaves between traffic states, as well as acceleration fans.

1. Introduction

Lighthill–Whitham–Richards (LWR) theory or kinematic wave theory, introduced by Lighthill and Whitham (1955) and Richards (1956), consists of two main equations: the conservation of vehicles and the equilibrium flow-density relationship. Assuming that traffic is always in an equilibrium state, these combine into a single partial differential equation for the propagation of traffic along a network link. Traditionally, this partial differential equation has often been solved by the Cell Transmission Model (CTM) (Daganzo, 1994), that discretizes roads into small cells according to the Godunov (1959) scheme. The Lagged Cell Transmission Model (LCTM) (Daganzo, 1999) and its later enhancement (Szeto, 2008) are variants of this method, reducing the numerical error.

Newell (1993) proposed a very different solution scheme, using cumulative numbers of vehicles as the primary variable. Later, this idea led to the development of the Link Transmission Model (LTM) (Yperman et al., 2006; Yperman, 2007), which does not discretize space and consequently leads to substantially smaller numerical errors (or computation time) than both the CTM and the LCTM. Daganzo (2005a,b) and Jin (2015) show implicitly and Han et al. (2016) show explicitly that this numerical procedure indeed solves the partial differential equation as the time step tends to zero, but only for triangular fundamental diagrams (FDs).

However, the requirement of triangular FDs is restrictive in multiple ways. Edie (1961) already identified that the speed in subcritical traffic decreases as traffic density increases. It is important to capture this relationship for the modeling of travel times in light traffic, yet a triangular FD does not do so. A non-linear free-flow branch in the FD furthermore captures

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platoon dispersion (Geroliminis and Skabardonis, 2005). Edie also recognized that there may be a discontinuity between the free-flow capacity and the queue discharge rate. This is commonly referred to as the capacity drop, i.e. the effect that the presence of congestion reduces the maximum flow. Papageorgiou (1998) mentioned this as an important aspect of traffic flow that models should be able to reproduce, particularly when considering traffic control. This is especially relevant when testing or optimizing traffic control measures aimed at preventing or postponing the occurrence of the capacity drop, like many ramp metering installations, or at dissolving stop-and-go waves or wide moving jams, like SPECIALIST (Hegyi et al., 2008). While Hajiahmadi et al. (2013) propose an extension to the LTM for variable speed limits and ramp metering, the lack of a capacity drop in triangular FDs thus significantly restricts its usability, e.g. in assessing control strategies. Separate modeling of a free-flow capacity and a queue discharge rate can furthermore be expected to benefit strategic assessments of intelligent in-vehicle systems designed to intervene specifically in case of congestion, such as the Congestion Assistant (Van Driel and Van Arem, 2010).

Unlike the continuous-space LTM, these issues have mostly been addressed for the discrete-space models. General continuous FDs can be handled by the CTM and LCTM with proven convergence to LWR theory (Daganzo, 1995; Daganzo, 1999; Szeto, 2008) and have been incorporated into CTM-based optimization problems (Nie, 2011; Carey and Watling, 2012). Multiple different modifications of the CTM have been proposed to deal with a capacity drop (see Section 4). None of this has so far been the case for the LTM.

Hence, the purpose of this paper is to overcome the aforementioned limitations of the shape of the FD in the LTM. More specifically, we extend the LTM to handle general concave FDs, optionally including capacity drops. The resulting model, which we show to converge to LWR theory if there is no capacity drop, is applicable to general networks and features both standing queues, with a head initially fixed at the bottleneck that may move upstream later, and stop-and-go waves that can both grow and dissolve. Qualitative properties of this model are demonstrated with numerical examples.

This paper is structured as follows. First, Section 2 will briefly introduce the structure of the LTM, consisting of a link model and a node model. Then, in Section 3, we derive a link model algorithm for the general case of a continuous concave FD, proving its consistency with LWR theory, and compare it to other link model formulations previously proposed in literature. Next, in Sections 4–6, we review previous work extending first-order models with capacity drops and subsequently extend LWR theory, the previous link model and a node model to allow for a capacity drop. Section 7 demonstrates the final model with two numerical examples. Finally, we list our conclusions in Section 8.

2. Structure of the LTM

The starting point of this paper is the LTM for dynamic network loading, as formulated by Yperman et al. (2006) and Yperman (2007). Its primary components are a link model and a node model, that together are used to update in time steps of size Δt the cumulative number of vehicles $N(x, t)$ at the entrance $x_{i,0}$ and exit $x_{i,L}$ of each link i.

The link model is used to determine the sending and receiving flows, which for each time step indicate the number of vehicles that could potentially exit and enter the link respectively. For link i, these quantities are denoted $S_i(t)$ and $R_i(t)$ respectively. The procedures for determining them are similar and will be discussed in more detail below.

The node model then considers the interactions of traffic at intersections to derive transition flows $G_{ij}(t)$, the number of vehicles that succeed in crossing the intersection, where different node models can be used to represent different types of intersections. They indicate how much of each turn-specific sending flow $S_{ij}(t)$ will actually pass the node during the time step.

The node model also needs to know the turning fractions $S_{ij}(t)/S_i(t)$ as input, which can be either specified exogenously or modeled endogenously by splitting the traffic into multiple so-called commodities with different routing behavior. Yperman et al. (2006) and Yperman (2007) assumed the latter option in their model formulation. Although we do not consider the specification of turning fractions in this paper, the results of this paper can be used in both these cases.

Algorithm 1 summarizes the overall process, showing how the link model and the node model together specify the traffic flow propagation.

The LTM discretizes only time, not space. Because due to the Courant-Friedrichs-Lewy (1928) condition, the maximum possible time step of a node depends on the length of the attached links, Yperman (2007) suggested that different nodes may be operated with different time step sizes to retain a high computational efficiency without being restricted by the smallest link in a (large) network. In this paper we explicitly incorporate this suggestion by writing Δt_{x_0} and Δt_{x_L} for the

Algorithm 1 Link Transmission Model.

- For each time step t for each node:
    - Using the link model, determine sending flow $S_i(t)$ for each incoming link i.
    - Using the link model, determine receiving flow $R_j(t)$ for each outgoing link j.
    - Determine turning fractions $\frac{S_{ij}(t)}{S_i(t)}$ for each turn ij.
    - Using a node model, determine transition flows $G_{ij}(t)$ for each turn ij.
    - $N(x_{i,L}, t+\Delta t) := N(x_{i,L}, t) + \sum_j G_{ij}(t)$ for each incoming link i.
    - $N(x_{j,0}, t+\Delta t) := N(x_{j,0}, t) + \sum_i G_{ij}(t)$ for each outgoing link j.