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Optimal queue placement in dynamic system optimum solutions for single origin-destination traffic networks

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ABSTRACT

The Dynamic System Optimum (DSO) traffic assignment problem aims to determine a time-dependent routing pattern of travellers in a network such that the given time-dependent origin-destination demands are satisfied and the total travel time is at a minimum, assuming some model for dynamic network loading. The network kinematic wave model is now widely accepted as such a model, given its realism in reproducing phenomena such as transient queues and spillback to upstream links. An attractive solution strategy for DSO based on such a model is to reformulate as a set of side constraints apply a standard solver, and to this end two methods have been previously proposed, one based on the discretisation scheme known as the Cell Transmission Model (CTM), and the other based on the Link Transmission Model (LTM) derived from variational theory. In the present paper we aim to combine the advantages of CTM (in tracking time-dependent congestion formation within a link) with those of LTM (avoiding cell discretisation, providing a more computationally attractive with much fewer constraints). The motivation for our work is the previously-reported possibility for DSO to have multiple solutions, which differ in where queues are formed and dissipated in the network. Our aim is to find DSO solutions that optimally distribute the congestion over links inside the network which essentially eliminate avoidable queue spillbacks. In order to do so, we require more information than the LTM can offer, but wish to avoid the computational burden of CTM for DSO. We thus adopt an extension of the LTM called the Two-regime Transmission Model (TTM), which is consistent with LTM at link entries and exits but which is additionally able to accurately track the spatial and temporal formation of the congestion boundary within a link (which we later show to be a critical element, relative to LTM). We set out the theoretical background necessary for the formulation of the network-level TTM as a set of linear side constraints. Numerical experiments are used to illustrate the application of the method to determine DSO solutions avoiding spillbacks, reduce/eliminate the congestion and to show the distinctive elements of adopting TTM over LTM. Furthermore, in comparison to a fine-level CTM-based DSO method, our formulation is seen to significantly reduce the number of linear constraints while maintaining a reasonable accuracy.

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1. Introduction

The Dynamic System Optimum (DSO) traffic assignment problem predicts the optimal time-dependent routing pattern of travellers in a network such that the given time-dependent origin-destination demands are satisfied and the total system travel time spent by all travellers is minimized, assuming some model for dynamic network loading. It has been shown recently by Shen and Zhang (2014) that DSO may have multiple solutions which share the same DSO objective (i.e. the total travel times) but have different queue lengths at nodes in the network. There are two types of special DSO solutions: the non-holding back solutions and the free-flow solutions. Non-holding back solutions require the system to discharge flow as much as it can so that there might be heavy congestion at the intersections inside the network, which in many cases might cause spillbacks to the upstream links. Here the spillback appears when downstream congestion in a link propagates upstream and reaches the upstream node, and hence possibly restricts the inflow to the link, and subsequently to restrict the exit flow from adjacent upstream links (Qian et al., 2012). The free-flow DSO solutions does not allow vehicles to wait in the queue inside the network, but allows the queue to build up at the origin nodes. Given the contrast in where the queues form in the network, the interesting question to ask is how to optimally distribute queues inside the network in order to achieve a better DSO solution at the end. This paper focuses on the DSO solutions which optimally distribute the (horizontal) congestion over links in the network. We expect that such optimal distribution of the congestion over links will lead to better traffic operation inside the network such as minimal queue spillbacks or even free-flow condition, which consequently reduce the stop-start traffic dynamics in the network and, hence, reduce emissions. To this end, there is a need to explicitly determine the time and space evolution of the queues in the network.

There has been much effort in literature undertaken to formulate the kinematic wave model (KWM) of Lighthill and Whitham (1955) and Richards (1956), based on an analogy between traffic flow and certain types of wave motion in fluids, for the DSO problem. Especially, a discrete version of the KWM, called the cell transmission model (CTM) (Daganzo, 1994; 1995) has been formulated as side constraints in the DSO problem (Carey and Ge, 2012; Carey and Watling, 2012; Gentile et al., 2005; 2007; Lo and Szeto, 2002; Nie and Zhang, 2005; Shen and Zhang, 2014; Szeto et al., 2011; Szeto and Lo, 2006; Ukkusuri et al., 2012; Ukkusuri and Waller, 2008; Ziliaskopoulos, 2000). It is because knowing the time-space dynamics of traffic flow within the link will facilitate a better understanding of the resulting SO solution, for example the level of congestion and spillback location in the network, etc. An important property of the CTM is the possibility to reformulate it as a relaxed set of linear constraints so that a linear programming model for the DSO-DTA problem for a network can be solved efficiently (Beard and Ziliaskopoulos, 2006; Li et al., 2003; Ukkusuri and Waller, 2008; Ziliaskopoulos, 2000). However, the choice to adopt CTM is not without its computational overheads. As noted in Nie and Zhang (2005), if we just consider the issue of DNL (of given route in-flow profiles) then the computational time for the CTM is directly proportional to the number of cells, and hence the computational efficiency is inversely proportional to the accuracy (in recovering the LWR). Thus the choice of discretisation level effectively means a choice/compromise between computational efficiency and the level of agreement with the (continuum) KWM model. If we then consider the wider issue of how the model is integrated within a DSO framework, then we are faced with further computational issues: if the CTM is specified as a set of side constraints as suggested by Peeta and Ziliaskopoulos (2001) then the number of constraints grows with the fineness of the discretisation, whereas if we represent it using a route-based mapping as in Lo and Szeto (2002), then we are led down a path of route enumeration with all the computational difficulties that it is known to bring. As noted in Bar-Gera (2005), the 'computational requirements reduce the attractiveness of this model for large-scale long duration applications'. Such computational issues of CTM based DSO problems are further explored in Shen and Zhang (2008) for different network sizes, where the number of constraints is increased polynomially with the network size.

Recently, a well known Lax–Hopf (LH) formula for the Hamilton–Jacobi (HJ) type partial differential equations has been used to solve the KWM, which can avoid the discretisation, hence enhance the computational efficiency. The LH formula can be used to provide a variational formulation of the HJ equation solutions describing the evolution of cumulative number of vehicles at the two ends of the link. In principle, the LH formula has been derived in various ways including the traffic flow theory by Daganzo (2005); 2006) which actually generalizes the theory of Newell (1993), the viability theory by Aubin et al. (2008); 2011) for given boundary conditions at two ends of the link, which is then extended to include the internal condition (i.e. information of probe vehicles) by Claudel and Bayen (2010a, 2010b), and the technique of calculus of variations by Evans (2010). The Link Transmission Model (LTM) in either discrete form (Yperman et al., 2005) or continuous form (Han et al., 2015; Jin, 2015) has been developed using the Newell's theory which is a special formulation of LH formula above, where the state of the whole link (i.e. either free-flow or congested) will be determined by the entry and exit flow. More specifically, the flow propagation in LTM is based on Newell's cumulative flow curves applied at the entry/exit of each link, with node models used to calculate the transition flows, which are based on conservation of flow between the incoming and outgoing flows. Sending/receiving flows, together with transition flows and other flow constraints, form the basis for updating the cumulative flows at the link boundaries. Osorio and Flotterod (2014); Osorio et al. (2011) have then developed another version of LTM, which is a so-called Double Queue Model (DQM). In the DQM, the link is treated as a set of two queues, referred to as the upstream queue and the downstream queue. Both LTM and DQM can properly capture the free-flow travel time delay when the link is in a free-flow state and the backward shockwave time delay when the link is in a congested state, which make them possible to capture queue spillbacks. The DQM was used in Ma et al. (2014) to find a free-flow DSO solution where spillback is tracked by the traffic state at the link entrance (in free flowing) or at the link exit (in congested) accounting for some time shift. Nevertheless, either LTM or DQM does not determine explicitly