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An efficient iterative link transmission model

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ABSTRACT

In this paper a novel iterative algorithm is presented for the link transmission model, a fast macroscopic dynamic network loading scheme. The algorithm's solutions are defined on a space–time discretized grid. Unlike previous numerical schemes there is no hard upper limit on the time step size for the algorithm to be numerically stable, leaving only the trade-off between accuracy and interpolation errors. This is a major benefit because mandatory small time steps in existing algorithm (required for numerical tractability) are undesirable in most strategic analyses. They lead to highly increased memory costs on larger network instances and unnecessary complex behaviour. In practice results are often aggregated for storage or analysis, which leads to the loss of computationally expensive detailed information and to the introduction of inconsistencies. The novel iterative scheme is consistent with the modelling assumptions independent of the numerical time step. A second contribution of the iterative procedure is the smart handling of repeated runs, which can be initialized (or warm started) by an earlier solution. For applications, repeatedly loading a network is often needed when evaluating traffic states under changing variables or adjusted parameter settings, or in optimization and equilibration procedures. In these cases the iterative algorithm is initialized with the solution of a previous run and iterations are performed to find a new consistent solution. Pseudo-code is provided for both a basic upwind iterative scheme and an extended algorithm that significantly accelerates convergence. The most important computational gains are achieved by ordering and reducing calculations to that part of the network which has changed (most). The properties of the algorithm are demonstrated on a theoretical network as well as on some real-world networks.

1. Introduction

Dynamic network loading (DNL) models are designed to represent the propagation of traffic over time in road networks. They provide an accurate description of network performance including: saturation, time varying travel speed, the location of bottlenecks, queue formation/dissipation and blocking back due to upstream spillback propagation. A substantial amount of research has been carried out in the last decades. For an overview on the field of dynamic traffic flow models the reader is referred to the extensive classification of Wageningen-Kessels et al. (2014). This overview focuses on the description of traffic flow on links. Equally important for DNL is the role of intersections and their representation as nodes in the network, connecting boundaries of links and imposing capacity constraints themselves; for an overview of node models for DNL,

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see Tampère et al. (2011). The focus here is on macroscopic traffic simulations which have the underlying assumption that traffic can be described as a continuum and compute the evolution of aggregated traffic characteristics such as density, flow or average speed. For many applications (strategic optimization, policy planning, dynamic traffic management, etc.) a simulation tool that provides an aggregated solution is sufficient and it is computationally more efficient than more detailed methods such as microscopic (car following) models.

This research frames within the class of DNLs referred to as Link Transmission Models. It is a macroscopic DNL building on Newell's simplified theory of kinetic waves (Newell, 1993a) and was first introduced by (Yperman et al., 2005). Link transmission models track traffic in a space–time discretized grid, for which they exploit the largest possible spatial discretization based on the longitudinal homogenous characteristics of a road or link – hence their name. However as discussed later in depth, temporal discretization in classical algorithms of LTM is often limited to small time intervals (∼ a few seconds). This is purely for computational reasons, as for analysis, such results are usually aggregated to larger time intervals (∼ multiple minutes). The first contribution of this paper is a method for handling sparse solution grids (with large time intervals), while still providing an accurate description of congestion patterns and the evolution of traffic states. A second contribution is an extension and optimization of this algorithm, herewith offering techniques for re-evaluating a scenario with altered input variables or parameters with a minimum of calculations. Such repeated calculations on the same network with only marginal changes (Corthout et al., 2014) are often required in practice for the computation of equilibrium routing behavior; the calibration of demand and supply; the optimization of network design, pricing instruments or control measures; the analysis of perturbations in robustness and reliability analysis.

In the next section, a short overview is presented of the basic theoretical principles. On the link level, first order kinematic wave theory and the fundamental diagram are introduced to elaborate an analytical solution for a single homogenous link. Subsequently, links in a network are connected by incorporating node models and introducing a computational grid. In Section 3, a theoretical network is presented to illustrate the principles of the algorithm and motivate the core elements. First, the classical upwind scheme of the LTM is converted to an iterative approach such that large time intervals are made possible. Next, the initialisation of the iterative procedure is presented and its importance in the handling of repeated calculations, continued by the introduction of a node ordering strategy to speed up convergence. Computational efficiency is subsequently further increased by reducing redundant calculations to a minimum. This also provides significant increases in performance for repeated network loadings. The complete calculation scheme or algorithm is presented in Section 4. The algorithm is capable of simulating traffic on a general (large-scale) network for which the time varying demand and route choice is given as an exogenous input. Pseudo-code is provided for both a basic variant, limited to the iterative procedure, and an extensive variant, which includes all the speed-ups. In Section 5 the algorithm is tested on some real world networks with different morphologies and sizes. The result for different time intervals and speed-ups is investigated by comparing the congestion patterns and travel times as well as the computation times. The case studies are concluded by applying the algorithm to scenarios that require repeated calculations to illustrate the potential computation gains in practice. The paper is concluded with a short discussion on the contributions of the new scheme and its relevance.

2. Theoretical background

2.1. First order kinematic wave theory for a single homogenous link

Many approaches to the macroscopic, analytical descriptions of traffic on a single homogenous road have been developed over the years. Its theory and impact on DNL and DTA literature are summarized next, to quickly converge to the elements that are important for the development of the link model for our network model. The fundaments of kinematic wave theory for traffic have been described by LWR-theory (Lighthill and Whitham, 1955; Richards, 1956) more than half a century ago. It is composed of a partial differential equation (PDE) that represents the conservation of vehicles, and a fundamental relation (Fig. 1) between instantaneous flow and local density called the fundamental diagram (FD). The LWR-PDE is described in function of the density of vehicles along a road. Classical first order finite difference approaches (Godunov, 1959) for numerically solving PDEs are widely used in practice, e.g. CTM of Daganzo (1994) or Lebacque (1996). These methods operate on a computation grid that discretizes space in cells and time in slices to provide an approximate solution to the PDE. The solutions have interesting physical properties like storage of vehicles (in terms of vehicle densities) along a link, the formation of shock waves and the relations between traffic states along characteristic speeds such as observed on real roads. However, the method is also computationally demanding requiring a link to be cut into smaller cells and updating times to be limited by the Courant Friedrich and Lewy (CFL) conditions (Courant et al., 1967). These CFL-conditions state that the time discretization should not be longer than the shortest possible travel time of traffic over a cell. With short cells and high speeds, this time step is in realistic networks typically in the order of a few seconds. In this paper, we provide a numerical scheme that circumvents these limitations.

This work builds on the seminal work of Newell on simplified theory of kinematic waves (Newell, 1993a, 1993b, 1993c). He simplified the FD to a triangular shape that proves to be efficient for numerical schemes in realistic networks (Yperman et al., 2005), and more importantly, he used cumulative vehicle number (CVN) functions to represent traffic. This intermediate abstraction (Moskowitz, 1965) by CVN functions (referred to as $N(x, t)$ at location x and time instant t) has also been of great importance for building exact solutions. It was formalized by Claudel and Bayen (2010b), Daganzo (2005b), Friesz et al. (2013) and Laval and Leclercq (2013) using theory of variations and Hamilton Jacobi (HJ) PDEs for general FD. Solutions