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An efficient and exact event-based algorithm for solving simplified first order dynamic network loading problems in continuous time

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ABSTRACT

In this paper a novel solution algorithm is proposed for exactly solving simplified first order dynamic network loading (DNL) problems for any generalised network. This DNL solution algorithm, termed eLTM (event-based Link Transmission Model), is based on the seminal Lighthill–Witham–Richards (LWR) model, adopts a triangular fundamental diagram and includes a generalised first order node model formulation. Unlike virtually all DNL solution algorithms, eLTM does not rely on time discretisation, but instead adopts an event based approach. The main advantage of this approach is the possibility of yielding exact results. Furthermore, an approximate version of the same algorithm is introduced. The user can configure an a-priori threshold that dictates the approximation error (measurable a-posteriori). Using this approximation the computational effort required decreases significantly, making it especially suitable for large scale applications. The computational complexity is investigated and results are demonstrated via theoretical and real world case studies. Fixed periods of stationary demands are included adopting a matrix demand profile to mimic basic departure time demand fluctuations. Finally, the information loss of the approximate solution is assessed under different configurations.

1. Introduction

The advances in traffic assignment models have progressed slowly but steadily over the last 60 years. While traditional static models, without an explicit time dimension (e.g., Beckmann et al., 1956) are still widely used, dynamic models have gained more and more attention in the past two decades. The main reason to move away from traditional static models is because they are known for not representing traffic in a realistic way. On the other hand, their well understood mathematical properties, stability, simplicity and – arguably the most important reason – computationally friendly nature have made them a popular choice to this very day. This shows that a theoretical superior model by itself is not enough to be adopted in practice; complexity and efficiency of a model are important factors as well.

Since the early 1990s, computational power allowed for more advanced models and the first steps towards practical dynamic network models were made. In this paper we focus on such dynamic models. To date, most dynamic models are still

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severely hindered by their large run times, often forcing practitioners to utilise simpler models than desired. Any contribution that either reduces computation time or improves the accuracy of dynamic models is therefore not only relevant in an academic sense, but can make a difference in the real world as well.

In the literature this group of models is adopted within the dynamic traffic assignment (DTA) paradigm. DTA incorporates both a flow propagation model, better known as a dynamic network loading (DNL) model, and a route choice model. Over the years the DNL model component branched off into three categories, namely microscopic, mesoscopic, and macroscopic approaches (see Peeta and Ziliaskopoulos, 2001). For highly detailed, often smaller scale networks, microscopic DNL models are often adopted. While potentially the most realistic, they can lack rigorous mathematical formulations and are often unstable due to the stochastic nature of the micro simulation, requiring many runs to get some form of 'representative' outcome. On the opposite end of the spectrum macroscopic models reside, adopting traffic flow theory in order to model traffic as a fluid (i.e., a continuous stream of vehicles). These models are traditionally utilised for larger scale networks. Typically free of random components, macroscopic models are easier to configure and produce more stable outcomes. However, they can lack some of the realism of their microscopic counterparts, especially in an urban setting. The third category – mesoscopic models – adopt characteristics of the two others, often modelling vehicles individually, but propagating them according to an underlying fundamental diagram (Mahut, 2000). In this paper we consider a macroscopic approach to DNL.

All three categories of DNL models can be subdivided into first or higher order models. First order models are capable of describing most elemental traffic characteristics such as queue formation and spillback and as such have been widely used in DTA models. While first order models describe steady-state traffic conditions and assume instantaneous speed changes, higher order models are able to explicitly incorporate acceleration and deceleration effects caused by traffic state transitions. Although these higher order models can explain hysteresis, capacity drop, stop-and-go waves, etc., they have also been criticised (e.g., Daganzo, 1995a; Papageorgiou, 1998). In this paper only first order models are considered.

The first order DNL problem is typically formulated using a link model and a node model. On the link level, the general kinematic wave model of Lighthill and Whitham (1955) and Richards (1956), better known as the LWR model, is considered. At the node level, a generalised first order node model formulation is adopted. We propose a novel algorithm to solve the (continuous-time) simplified first order macroscopic DNL problem for general networks. Three assumptions are made: (i) A triangular fundamental diagram is assumed, (ii) the travel demand is assumed stationary per time period (which holds for virtually all model applications in practice), and (iii) route choice is assumed to be pre-trip and applied through splitting rates at the node level that are kept fixed during each time period.

We term this new solution approach the event-based Link Transmission Model (eLTM). By utilising a continuous-time formulation in which only changes in flow rates are propagated through the network, the need for any time discretisation is obviated (it is well known that time discretisation often leads to averaging errors). The proposed algorithm is capable of computing an exact solution. The innovation in this approach is found in its efficiency; the algorithm is able to construct cumulative inflow and outflow curves by only tracking the exact moments the flow rates change. This benefit is further exploited by the assumption that travel demand in each time period (typically between five minutes to one hour) is temporarily stationary, which leads to link flows becoming temporarily stationary. Many macroscopic models use simulation time steps of one second, but if the link inflows and outflows do not change every second, it is computationally more efficient to only update the link inflows and outflows when needed. In case there are many (small) flow changes, an optional flow change acceptance threshold is proposed. This threshold can eliminate the propagation of minor flow changes through the network, significantly reducing the computational cost of the algorithm. However, this increased efficiency comes at the cost of a non-exact solution. Therefore, the main contributions of this paper are twofold, namely (i) an algorithm that is able to calculate exact solutions on general networks of the simplified first order DNL problem (using a zero threshold), this in contrast to currently existing non-exact methods, and (ii) an algorithm that, in most cases, is (much) more efficient than currently existing methods (using a non-zero threshold). Since the focus of this paper is on DNL, path demand fluctuations are assumed exogenous and modelled via a basic demand profile.

The structure of the paper is as follows. In Section 2 we briefly summarise the current state-of-the-art in first order DNL models and algorithms, and show that existing methods cannot compute exact solutions in general transport networks. Although link and node models are not a contribution of this paper as such, we reformulate existing models in Sections 3 and 4 in order to present our novel solution algorithm in Section 5. Section 5 also outlines the computational complexity of the proposed algorithm and is compared to existing models. In Section 6 we demonstrate the validity of our algorithm on two simple hypothesised corridor networks, and show feasibility on a large real-world network. Section 7 provides a final discussion and conclusion.

2. Macroscopic dynamic network loading models

Most macroscopic DNL models are underpinned by kinematic wave theory. This theory entails the way (shock) waves of traffic traverse a road and is first described by Lighthill and Whitham (1955) and Richards (1956). Let $q(x, t)$ and $k(x, t)$ be the flow rate (veh/h) and density (veh/km) at location x at time instant t, respectively. The conservation of flow defines the basic relationship between how a change in density over time relates to a change in flow rate over space in such a way that no vehicles can be lost in the process, leading to the well-known conservation law: