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A disjunctive convex programming approach to the pollution-routing problem

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ABSTRACT

The pollution-routing problem (PRP) aims to determine a set of routes and speed over each leg of the routes simultaneously to minimize the total operational and environmental costs. A common approach to solve the PRP exactly is through speed discretization, i.e., assuming that speed over each arc is chosen from a prescribed set of values. In this paper, we keep speed as a continuous decision variable within an interval and propose new formulations for the PRP. In particular, we build two mixed-integer convex optimization models for the PRP, by employing tools from disjunctive convex programming. These are the first arc-based formulations for the PRP with continuous speed. We also derive several families of valid inequalities to further strengthen both models. We test the proposed formulations on benchmark instances. Some instances are solved to optimality for the first time.

1. Introduction

Transportation accounts for a significant portion of Greenhouse Gas (GHG) emissions, e.g., 27% among all sources in the United States in 2013 (Environmental Protection Agency, 2015). Approaches to reduce GHG emissions in transportation include switching to cleaner fuels, improving fuel efficiency, reducing travel demand, etc. Another important approach is to improve operational practices. In classical operational models such as the vehicle routing problem (VRP), a common assumption is that the travel cost and travel time between two vertices is given as input data. The fuel consumption and GHG emissions, however, heavily depend on the routing and scheduling decisions. Many new transportation models have been proposed recently to improve fuel efficiency and reduce the negative environmental impact; see for instance recent surveys on green freight transportation (Demir et al., 2014b; Eglese and Bektaş, 2014; Lin et al., 2014).

Speed control plays a significant role on fuel consumption and GHG emissions. For instance, the fuel consumption as well as the GHG emissions per kilometers of a heavy-duty truck at a high speed can be almost twice as much as that at a lower speed (Bektaş and Laporte, 2011), and a computational study in Norstad et al. (2011) showed that optimizing speed on a ship route can reduce fuel consumption by as much as 14%. The integration of vehicle speed as a decision variable into traditional transportation models provides opportunities for savings in fuel consumption and reduction in GHG emissions. On the other hand, the dependence of fuel consumption and GHG emissions on speed is usually nonlinear in practice, im-

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posing great challenges on solving the resulting model exactly. In particular, the objective function and any constraint that involves travel duration in the classical routing models will now be nonlinear in speed. One common exact approach to handle this issue is by speed discretization, i.e., assuming the vehicle speed takes a discrete set of prescribed values within the limits. Then the nonlinear cost for every prescribed speed value can be computed and stored in advance, and the resulting model becomes a mixed-integer linear programming problem. One issue with speed discretization is that the true optimal speed may be excluded in advance from the prescribed set of speed values, if the chosen discretization is not fine enough. Another potential issue is that speed discretization introduces a large number of additional variables, which renders the model computationally challenging to solve for large-size instances. In this paper, we take a new approach by maintaining speed as a continuous decision variable. We introduce a formulation framework to directly incorporate the nonlinear relationship between cost and speed into the routing problem, by employing tools from disjunctive convex programming. In particular, we focus on the pollution-routing problem (PRP) proposed by Bektaş and Laporte (2011). The PRP aims to find a set of vehicle routes and vehicle speeds over the routes to minimize the operational and environmental costs, while respecting constraints on time and vehicle capacities. The PRP is NP-hard, since it generalizes the VRP. Even a medium-size PRP instance is extremely challenging to solve exactly: as shown in Bektaş and Laporte (2011), the mixed-integer linear programming formulation through speed discretization for a 15-city one-vehicle instance cannot be solved by CPLEX 12.1 to optimality within 3 h. Our paper makes the following contribution to the literature on models that jointly optimize routing and speed decisions.

- We introduce the first arc-based formulations for the PRP with continuous speed. Our formulations directly model the nonlinear relationship between fuel consumption and vehicle speed, without speed discretization. These formulations provide a benchmark for the maximum cost saving and emission reduction that can be achieved by speed optimization. In addition, the reformulation techniques from disjunctive convex programming can be applied to convex fuel consumption functions other than the one used in the PRP.
- We study the theoretical strengths of the two mixed-integer convex programs (MICPs) we propose. In particular, we compare the lower bounds provided by the continuous relaxations of the formulations. We also derive several new families of valid inequalities tailored for the PRP to strengthen these formulations.
- We test our formulations on benchmark instances (Bektaş and Laporte, 2011; Kramer et al., 2015b). Instances with up to 25 cities can be solved to optimality, with some instances solved to optimality for the first time.
- We analyze the impact of labor cost and departure time at the depot on the optimal routing decisions. We also compare the optimal solutions from our formulations with the optimal solutions through speed discretization. It shows that for the test instances the solutions from speed discretization always give the same optimal routes but not the same optimal speeds. The comparison provides some empirical justification on speed discretization for the PRP.

The rest of the paper is organized as follows. Section 2 reviews results related to the PRP in the literature. Section 3 gives a complete description of the PRP. Section 4 gives a brief overview of disjunctive convex programming techniques that we employ in formulating the PRP as an MICP. Section 5 presents two MICP formulations for the PRP, and compares the lower bounds given by their continuous relaxations. Several families of valid inequalities are introduced in Section 6. Extensive computational results and analysis are given in Section 7. We conclude and point out several future research directions in Section 8.

2. Literature review

Speed control has a great economic and environmental impact in transportation, in particular maritime transportation. The daily fuel consumption of a ship increases dramatically with the sailing speed (Notteboom and Vernimmen, 2009; Ronen, 2011). It is shown in Hvattum et al. (2013); Laporte (2016) that with a fuel price of 450 USD/tonne, a one percent worldwide reduction in fuel consumption would yield an estimated cost reduction of more than 1.2 billion USD, and a reduction of CO_2 emissions of 10.5 million tonnes. The impact of speed control on reducing GHG emissions and fuel consumption in maritime shipping have been studied extensively; See Cariou (2011); Kim et al. (2012); Kontovas and Psaraftis (2011); Lee and Song (2016) and the references therein. One problem closely related to the PRP is the tramp speed optimization problem (SOP) studied by Fagerholt et al. (2010) and Norstad et al. (2011), in which a ship aims to find the optimal sailing speed over each leg of a given route to minimize the total fuel consumption. An efficient iterative method, called the speed optimization algorithm (SOA), was developed to solve SOP exactly when the fuel consumption function is a convex function of the ship speed (Hvattum et al., 2013; Norstad et al., 2011). The SOA was modified by Demir et al. (2012) to solve the SOP with additional labor cost, and by Kramer et al. (2015a) to handle the SOP with departure time at the origin as an additional decision variable.

The PRP integrates speed and routing decisions to minimize the total operational and environmental cost. The PRP generalizes several variants of the VRP in the following way: with fixed speed over each arc, the PRP reduces to the energy minimizing vehicle routing problem (EMVRP) (Kara et al., 2007; 2008) with time-window constraints, and is equivalent to the classical VRP with time windows when vehicle curb weight is much larger than cargo weight (Fukasawa et al., 2016b); the PRP also generalizes the fuel consumption VRP proposed in Xiao et al. (2012). The PRP is difficult to solve exactly for large-size instances. In Bektaş and Laporte (2011), the speed over each arc is allowed to take ten prescribed values and the PRP is modeled as a mixed-integer linear program. The resulted model is solved by CPLEX for small-size instances. Heuristic