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Parametric search for the bi-attribute concave shortest path problem

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ABSTRACT

A bi-attribute concave shortest path (BC-SP) problem seeks to find an optimal path in a bi-attribute network that minimizes a linear combination of two path costs, one of which is evaluated by a nondecreasing concave function. Due to the nonadditivity of its objective function, Bellman's principle of optimality does not hold. This paper proposes a parametric search method to solve the BC-SP problem, which only needs to solve a series of shortest path problems, i.e., the parameterized subproblems (PSPs). Several techniques are developed to reduce both the number of PSPs and the computation time for these PSPs. Specifically, we first identify two properties of the BC-SP problem to guide the parametric search using the gradient and concavity of its objective function. Based on the properties, a monotonic descent search (MDS) and an intersection point search (IPS) are proposed. Second, we design a speedup label correcting (LC) algorithm, which uses optimal solutions of previously solved PSPs to reduce the number of labeling operations for subsequent PSPs. The MDS, IPS and speedup LC techniques are embedded into a branch-and-bound based interval search to guarantee optimality. The performance of the proposed method is tested on the mean-standard deviation shortest path problem and the route choice problem with a quadratic disutility function. Experiments on both real transportation networks and grid networks show that the proposed method reduces the computation time of existing algorithms by one to two orders of magnitude.

1. Introduction

1.1. Motivation

The shortest path problem plays a fundamental role in the fields of transportation science, telecommunications, logistics, etc. The classical shortest path problem aims at finding an optimal path with the minimum travel time. However, in practice, usually more than one attributes of paths influence the optimal routing decisions. Furthermore, a risk-averse decision maker in a stochastic network considers not only the expected travel time but also the path reliability, such as the travel time variability. To strike a balance between two different criteria, optimal bicriterion routing decisions need to be made. To this

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end, this paper studies a bi-attribute concave shortest path (BC-SP) model of the form: $\min\{h(\nu(p)) + \mu(p) : p \in P\}$, where h: $R \to R$ is a non-decreasing differentiable concave function, P is the set of loop-less paths from an origin to a destination and $(\nu(p), \mu(p))$ are nonnegative attributes of a path $p \in P$.

An important application of the BC-SP model is the mean-standard deviation shortest path (MSD-SP) problem (Khani and Boyles, 2015; Wu and Nie, 2011), where h is the square root function, ν_{ij} and μ_{ij} represent the variance and mean of the random travel time of a path p. The MSD-SP problem seeks to find a reliable path, which minimizes the sum of the mean and weighted standard deviation of path travel time (Noland and Polak, 2002; Shahabi and Boyles, 2015; Zeng et al., 2015; Zhang et al., 2016a).

Besides the MSD-SP problem, the BC-SP model also arises in multi-attribute transportation networks. For example, as pointed out by Mirchandani and Wiecek (1993), the transportation cost is usually a non-decreasing concave function of travel distance, and thus considering both travel time and transportation cost gives rise to a BC-SP problem. Other applications of the BC-SP model include the traffic equilibrium problem (Gabriel and Bernstein, 1997; Zhang et al., 2011), the multi-agent network problem (Gabriel and Bernstein, 2000) and the nonlinear congestion pricing problem (Lawphongpanich and Yin, 2012).

The BC-SP problem belongs to discrete concave minimization problems, which are usually hard to solve exactly (Tuy, 1998). Due to its nonadditivity, Bellman's principle of optimality does not hold for the BC-SP problem, i.e., subpaths of optimal paths are not necessarily optimal. In the remainder of this section, we first review existing solution methods, and then outline the proposed method and our contributions.

1.2. Literature review

1.2.1. Enumeration methods

The monotonicity of the objective function indicates that optimal paths of the BC-SP problem are non-dominated (Pareto) paths. A direct method to solve the BC-SP problem is to generate all non-dominated paths and select the optimal one. Enumerating all non-dominated paths has been extensively studied as the bicriterion short path (BSP) problem since the seminal work of Hansen (1980). Solution methods include the label setting (LS), label correcting (LC), ranking and two phase methods. LS and LC methods generalize Dijkstra's and Bellman-Ford-Moore's shortest path algorithms to the multi-criteria setting, respectively. Most recent studies focus on speeding up labeling methods using dominance relations, such as Iori et al. (2010) (LS), Raith (2010) (LS, LC) and Demeyer et al. (2013) (LS). In addition, Lozano and Medaglia (2013) and Duque et al. (2015) propose a depth-first labeling method, i.e., the pulse algorithm. Ranking method utilizes k-shortest path (Climaco and Martins, 1982) or near shortest path algorithms (Raith and Ehrgott, 2009). Two phase methods first generate the extreme non-dominated paths using a parametric analysis and then generate the remainders by LS, LC or k-shortest paths algorithms (Mote et al., 1991). There are numerous studies on the BSP problem. A comprehensive overview is beyond the scope of this paper. The reader is referred to recent review papers, including Climaco and Pascoal (2012); Raith and Ehrgott (2009) and Paixao and Santos (2013).

The concavity of the objective function further shows that optimal paths of the BC-SP problem belong to the extreme non-dominated path set, which is a subset of the non-dominated path set. Therefore, it is sufficient to enumerate the extreme non-dominated paths. Henig (1986) proposes three methods to generate all extreme non-dominated paths: the sequential parametric analysis (SPA), the bidirectional parametric analysis (BPA) and dynamic programming (DP). The SPA method is a customized parametric linear programming method, which iteratively increases a parameter. The BPA method updates the parameter from both left and right directions. DP uses a generalized Bellman's principle of optimality. Recently, Sedeno-Noda and Raith (2015) improve the SPA method and propose a Dijkstra-like method computing all extreme non-dominated paths. To solve the MSD-SP problem, Khani and Boyles (2015) propose two iterative labeling (IL) algorithms: the basic one is similar to the SPA method and the improved one reduces the computation time by partially updating labels.

1.2.2. Implicit enumeration methods

Although enumeration methods can solve the BC-SP problem, Hansen (1980) shows that the number of non-dominated paths may grow exponentially in n, and Carstensen (1983) further shows that the number of extreme non-dominated paths could be $2^{\Omega(\log^2 n)}$, where n is the number of nodes. One approach to reduce the number of enumerations is to exploit dominance properties. Hutson and Shier (2009) propose a modified labeling algorithm based on an e-dominance condition for a general bicriterion convex-concave shortest path problem. Chen et al. (2013) propose multi-criteria LS and A* algorithms based on a fast dominance check for the MSD-SP problem.

Another implicit enumeration method is the parametric search method, which has been widely used to solve discrete optimization problems (Tuy, 1998). The parametric search method is similar to the BPA method, but it only needs to enumerate partial extreme non-dominated paths by solving a series of parameterized subproblems (PSPs): (P_λ) $\min\{\lambda\nu(p) + \mu(p) : p \in P\}$ where $\lambda \in [0, +\infty)$. Henig (1986) suggests a branch-and-bound (B&B) based parametric search to find the optimal non-dominated extreme path. To find the best non-dominated path through an interaction with decision makers, Current et al. (1990) and Coutinho-Rodrigues et al. (1999) combine the parametric search with constrained shortest path and k-shortest paths approaches. Xie and Waller (2012) propose a polynomial time parametric algorithm to approximate non-dominated paths of the BSP problem. Nikolova (2009) uses the parametric search to solve the BC-SP problem, which generates new parameters using the perpendicular method (Dial, 1979; Henig, 1986). Zhang et al. (2016b) further propose a B&B based