Transportation Research Part B 94 (2016) 189–203

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

Finding the *k* reliable shortest paths under travel time uncertainty

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ARTICLE INFO

Article history:
Available online 5 October 2016

Keywords:
K reliable shortest paths problem
Travel time uncertainty
Reliability

ABSTRACT

This paper investigates the problem of finding the K reliable shortest paths (KRSP) in stochastic networks under travel time uncertainty. The KRSP problem extends the classical K loopless shortest paths problem to the stochastic networks by explicitly considering travel time reliability. In this study, a deviation path approach is established for finding K α-reliable paths in stochastic networks. A deviation path algorithm is proposed to exactly solve the KRSP problem in large-scale networks. The A* technique is introduced to further improve the KRSP finding performance. A case study using real traffic information is performed to validate the proposed algorithm. The results indicate that the proposed algorithm can determine KRSP under various travel time reliability values within reasonable computational times. The introduced A* technique can significantly improve KRSP finding performance.

1. Introduction

The problem of finding the shortest path in road networks has been widely studied (Dijkstra, 1959; Bast et al., 2007; Geisberger et al., 2012; Zhou et al., 2014; Li et al., 2015), due to its broad application in transportation science and other fields (e.g., route guidance systems, traffic simulations, or logistics optimizations). As a natural generalization, the problem of finding K shortest paths has also received considerable attention in the literature (Hoffman and Pavley, 1959; Yen, 1971; Eppstein, 1998; Martins and Pascoal, 2003; Hershberger et al., 2007; Vanhove and Fack, 2012). Given an integer $K \geq 1$, the K shortest paths problem is intended to successively find the shortest path, the second shortest path, etc., until the K^{th} shortest path between an origin and a destination (O-D) pair. Provision of multiple alterative paths is a common requirement for route guidance systems, and widely used for solving optimization problems with complex constraints and/or multiple objectives. Typical applications of the K shortest paths problem are summarized in Eppstein (1998).

In the classical K shortest paths problem, link travel times are assumed to be deterministic. However, link travel times in road networks are widely recognized by transportation practitioners and researchers to be highly stochastic, due to demand fluctuation and capacity degradation (Jiang and Szeto, 2016; Tan et al., 2014; Lam et al., 2008; Yang and Zhou, 2014; Chen et al., 2013a; Miller-Hooks and Mahmassani, 2000; Fu and Rilett, 1998). Many empirical studies have shown that, faced with travel time uncertainty, travelers tend to become risk-averse when making activity-travel decisions (Beaud et al., 2016; Engelson and Fosgerau, 2016; Taylor 2013; Carrion and Levinson, 2012). Clearly, large travel time variations may cause

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http://dx.doi.org/10.1016/j.trb.2016.09.013
0191-2615/

undesirable late arrivals and impose a high penalty (e.g., missed flights). Therefore, travelers' concerns about travel time reliability should be incorporated into the shortest path problem, as well as the K shortest paths problem.

In recent years, much research effort has been devoted to investigating reliable shortest path problems for finding the optimal path by explicitly considering travel time reliability. Several efficient solution algorithms have been developed for solving the reliable shortest path problems in large-scale networks with uncertain travel times (Nie and Wu, 2009; Ji et al., 2011; Xing and Zhou, 2011; Chen et al., 2012; Chen et al., 2014a; Khani and Boyles, 2015; Wu, 2015). However, to the best of our knowledge, the problem of finding K reliable shortest paths (KRSP) has not received attention in the literature.

To fill the gap, this paper precisely addresses the KRSP problem. The rest of the paper is organized as follows. Section 2 provides a brief literature review related to the KRSP problem. Section 3 presents the formulation of the KRSP problem. Section 4 introduces the proposed solution algorithms for solving the KRSP problem. Section 5 reports a case study using real traffic information. Section 6 presents the conclusions and recommendations for further study.

2. Literature review

This section briefly reviews the related algorithms for solving the classical K shortest path problems and the reliable shortest path problems, to provide necessary background to the KRSP problem.

Hoffman and Pavley (1959) first investigated the K shortest paths problem. Since then, numerous algorithms have been developed for solving two variants of the K shortest paths problem. In the first variant, the K shortest paths are allowed to contain cycles (i.e., non-simple paths). For finding K non-simple paths, the best known algorithm is by Eppstein (1998) with worst case complexity $O(|A|+|N|Log|N|+K)$, where $|A|$ and $|N|$ are the number of links and nodes in the network, respectively. In the second variant, the K shortest paths are not allowed to have cycles (called simple or loopless paths). Compared to its non-simple counterpart, this variant of finding the K loopless paths is significantly harder. Yen's algorithm (Yen, 1971) is acknowledged as the best algorithm with lowest worst-case complexity $O(K|N|(|A|+|N|Log|N|))$. This algorithm is built on the deviation path concept established by Clarke et al. (1963), which is to find the K^{th} loopless shortest path from all paths deviating from the first $K-1$ loopless shortest paths. A complete survey of existing K shortest paths algorithms is out of the scope of this paper; interested readers can refer to Eppstein (1998) and Hershberger et al. (2007).

Reliable shortest path problems have been intensively studied recently. The reliable shortest path problems can be roughly classified into the most reliable path problem (Frank, 1969) and the α-reliable path problem (Chen and Ji, 2005). The most reliable path problem is to find the optimal path by maximizing the travel time reliability for a pre-determined travel time budget, while the α-reliable path problem is find the optimal path by minimizing the travel time budget for a pre-determined travel time reliability. Chen et al. (2013b) noted that these two problems apply to different route guidance scenarios. The α-reliable path problem is commonly used in pre-trip planning, where travelers have freedom to choose their departure times; whereas the most reliable path problem is used for in-vehicle routing with fixed travel time budget.

Due to their non-linear and non-additive problem structure, reliable shortest path problems cannot be solved by classical deterministic shortest path algorithms (e.g., Dijkstra's algorithm). Several effective solution algorithms have been developed to exactly solve reliable shortest path problems. Mirchandani (1976) presented a recursive algorithm to determine the most reliable path. However, the algorithm requires path enumeration and can only be applied to very small networks. Based on the first order stochastic dominance (FSD) condition, Nie and Wu (2009) proposed a multi-criteria label-correcting algorithm exactly solve the most reliable path problem and α-reliable path problem. To efficiently solve the α-reliable path problem, Chen et al. (2013b) established Mean-Budget dominance condition to reduce the number of generated FSD non-dominated paths. A multi-criteria A* algorithm was then developed for efficiently finding the α-reliable path in large-scale road networks. This multi-criteria A* algorithm was further extended to consider limited spatial correlations in road networks (Chen et al., 2012). To find the α-reliable path with complete spatial correlations, Xing and Zhou (2013) proposed a Lagrangian substitution algorithm using a sample-based formulation. Zeng et al. (2015) extended the sample-based formulation by representing complete spatial correlations using a Cholesky decomposition.

Along the line of previous work (Yen, 1971; Chen et al., 2013b), this paper investigates the KRSP problem for finding K α-reliable paths in stochastic networks. Since cyclic paths are generally not relevant for road users in most transportation applications, only loopless α-reliable paths are considered. This paper is a first step towards investigating the KRSP problem with the following specific contributions:

First of all, the KRSP problem is formulated in stochastic networks based on the α-reliable path concept. The formulated KRSP problem extends the classical K loopless shortest paths problem (Yen, 1971) to stochastic networks by explicitly considering travel time reliability. The classical K shortest paths problem can be regarded as a special case of the KRSP problem under the risk-neutral scenario. Such KRSP problem can be very useful for route guidance systems and many optimization problems with complex constraints or multiple objectives in stochastic networks.

Secondly, the deviation path concept is established for finding K α-reliable paths in stochastic networks. The deviation path concept (Clarke et al., 1963) has proven effective for finding K shortest paths with linear and additive problem structure. This deviation path concept is extended in this study for finding K α-reliable paths with non-linear and non-additive problem structure. The established deviation path concept provides an effective way to determine the Kth α-reliable path from deviation paths of the $K-1$ α-reliable paths.

Thirdly, exact solution algorithms are proposed to solve the KRSP problem. A deviation path algorithm (called DP-LS) is proposed for exactly finding the K α-reliable paths in large-scale road networks. The multi-criteria label-setting algorithm