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Preferences for travel time under risk and ambiguity: Implications in path selection and network equilibrium

Jin Qi[a,*], Melvyn Sim[b], Defeng Sun[c], Xiaoming Yuan[d]

[a] *Department of Industrial Engineering and Logistics Management, Hong Kong University of Science and Technology, Hong Kong*
[b] *NUS Business School, National University of Singapore, Singapore*
[c] *Department of Mathematics and Risk Management Institute, National University of Singapore, Singapore*
[d] *Department of Mathematics, Hong Kong Baptist University, Hong Kong*

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ABSTRACT

In this paper, we study the preferences for uncertain travel times in which probability distributions may not be fully characterized. In evaluating an uncertain travel time, we explicitly distinguish between *risk*, where the probability distribution is precisely known, and *ambiguity*, where it is not. In particular, we propose a new criterion called *ambiguity-aware CARA travel time* (ACT) for evaluating uncertain travel times under various attitudes of risk and ambiguity, which is a preference based on blending the Hurwicz criterion and Constant Absolute Risk Aversion (CARA). More importantly, we show that when the uncertain link travel times are independently distributed, finding the path that minimizes travel time under the ACT criterion is essentially a shortest path problem. We also study the implications on Network Equilibrium (NE) model where travelers on the traffic network are characterized by their knowledge of the network uncertainty as well as their risk and ambiguity attitudes under the ACT. We derive and analyze the existence and uniqueness of solutions under NE. Finally, we obtain the Price of Anarchy that characterizes the inefficiency of this new equilibrium. The computational study suggests that as uncertainty increases, the influence of selfishness on inefficiency diminishes.

1. Introduction

In real world transportation networks, travel times are almost always uncertain, which is found to be one the most important factors in the path selection decisions (Abdel-Aty et al., 1995). Individuals' preferences greatly depend on their knowledge about the uncertain travel time as well as their attitudes towards uncertainty. In transportation literatures, an uncertain travel time is often associated with a random variable with the known probability distribution. In other words, the traveler knows the exact frequency of travel time outcomes, and his/her preference relies on his/her risk attitude, that is usually characterized by taking an expectation over a disutility function (an increase in the travel time amounts to a loss). Deliberating on reliability, Fan et al. (2005), Mirchandani (1976), and Nie and Wu (2009) consider the probability of punctuality as a preference criterion, which could be treated as a step disutility function. Unfortunately, since in general, computing the probability of a sum of random variables is NP-hard (Khachiyan, 1989), it is a computationally intractable problem to find the path with the minimum expected disutility over a transportation network, which severely limits our

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[*] Corresponding author.
E-mail address: jinqi@ust.hk (J. Qi).

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analysis and implementation. Murthy and Sarkar (1998) consider a piece-wise linear concave disutility function, and solve the problem with certain enumeration algorithms. Loui (1983) and Eiger et al. (1985) consider disutility functions in the form of linear, quadratic or exponential, in which the resultant static path selection problems are computationally tractable. In particular, De Palma and Picard (2005) justify empirically the relevance of the exponential disutility function, which appeals to travelers with Constant Absolute Risk Aversion (CARA) and has the best fit on path selection behavior amongst common disutility functions.

Implications of risk in Network Equilibrium (NE) problems, which model a collective behavior of a large population of travelers, have also been studied. One stream suggests using disutility function to capture travel time uncertainty, and travelers' attitudes towards risk (see Chen et al., 2002; Mirchandani and Soroush, 1987; Nagurney and Dong, 2002; Yin and Ieda, 2001; and Yin et al., 2004). The second stream discusses the travel time variability by adding the mean travel time with a safety margin, which can be described by a penalty function (see Noland and Polak, 2002; Watling, 2006), or the standard deviation (see Connors et al., 2007; Lo et al., 2006; Nikolova and Stier-Moses, 2014; Siu and Lo, 2008; Uchida and Iida, 1993). However, adding the safety margin in these ways may violate first-order stochastic dominance, and it generally cannot be separated by links, which makes the model hard to solve. We refer interested readers to the review papers of Noland and Polak (2002) and Connors and Sumalee (2009). We would like to distinguish our work with stochastic NE (see for instance, Sheffi, 1985). Stochastic NE model captures the variations in travelers' perception on the travel time but still assumes that the travel time is deterministic.

The assumption that travelers know the exact frequency of travel time outcomes is unrealistic. In a real world, it is conceivable that a traveler is incapable of knowing the entire probability distributions of the transportation network. Major exceptional events (e.g., natural disasters) and minor regular events (e.g., minor accident, traffic signal) will incur uncertainty to travel time. Hence, complete distribution of travel time is seldom known exactly, and even the estimated one could be considerably affected by the sampling procedure. If the actual travel time probability distribution is not fully known, then it would be impossible to establish the preferences for travel times based on the expected disutility criterion. In fact, the distinction between risk, where outcome frequency is known, and ambiguity, where it is not, can be retrospected to Knight (1921), and has been extensively studied in economics (see for instance Camerer and Weber, 1992; Gilboa et al., 2008; Mukerji and Tallon, 2004; Wakker, 2008), finance (see for instance Bossaerts et al., 2010; Chen and Epstein, 2002; Dow and Werlang, 1992; Epstein and Schneider, 2008; Guidolin and Rinaldi, 2013), and marketing (see for instance Erdem and Swait, 2007; Muthukrishnan et al., 2009). Ellsberg (1961) shows convincingly by means of paradoxes that ambiguity preference cannot be reconciled by classical expected utility theory. Inspired by this seminal work, numerous experimental and theoretical studies spring up to verify and accommodate this behavior issue. Notably, in Hsu et al. (2005) ground-breaking experiments, economists and neuroscientists collaborate to establish significant physiological evidence via functional brain imaging that humans have varying and distinct attitudes towards risk and ambiguity. The results also indicate that people's attitudes towards risk and ambiguity are not fully correlated, i.e., there exists a population of people that are ambiguity averse and risk-seeking, or ambiguity seeking and risk-averse.

From the normative perspective, ambiguity is also an active area of research within the domains of decision theory and operations research. Gilboa and Schmeidler (1989) consider ambiguity as a set of possible probability distributions, and present the Max-min Expected Utility (MEU) model, which appeals to ambiguity averse decision makers. To accommodate the heterogeneity of ambiguity and risk attitudes found in experiments, Ghirardato et al. (2004), based on Hurwicz criterion (Arrow and Hurwicz, 1972; Hurwicz, 1951), axiomatize the $\alpha-$MEU model, which represents a compromise via a convex combination of the worst and best case expected utility. The parameter α is an index of pessimism or optimism. However, the discussion on travel time ambiguity is relatively new. Yu and Yang (1998) propose a worst-case shortest path problem over a set of discrete scenarios, which results in an *NP*-hard problem. Bertsimas and Sim (2003) introduce the "budget of uncertainty" in characterizing uncertain travel time and show that the worst-case shortest path problem is a tractable optimization problem. Ordóñes and Stier-Moses (2010) extend the work to address an NE problem. They generally consider three cases of equilibrium with uncertain travel times: α-percentile equilibrium, added-variability equilibrium, and robust Wardrop equilibrium. The α-percentile equilibrium assumes travelers minimize the α quantile (or Value-at-Risk) of their experienced travel times, which are generally computationally intractable optimization problems. Added-variability equilibrium provides a safety margin to the expected travel time as a proxy to account for risk-averse behavior, an approach that may not be coherent with decision analysis such as violating first order stochastic dominance. Robust Wardrop equilibrium borrows the idea of Bertsimas and Sim (2003), and assumes that ambiguity averse travelers minimize the worst-case travel time given that the total variation is bounded by a certain parameter. However, the assumptions that the entire population of travelers are only ambiguity averse and not risk sensitive limit the application of this model.

In contrast to the aforementioned works that consider risk and ambiguity separately, our main contribution is to explicitly distinguish between risk and ambiguity in a unified framework in articulating travelers' preferences for travel times. We present a new criterion named *ambiguity-aware CARA travel time* (ACT) for evaluating uncertain travel times for travelers with various attitudes of risk and ambiguity. Apart from the behavioral relevance of the ACT, we also present a computational justification by showing that when the uncertain link travel times are independently distributed, finding the path that minimizes travel time under the ACT criterion is essentially a shortest path problem. We also study the implications on NE problem, in which travelers minimize their own travel times under the ACT criterion, and no traveler can improve his/her travel time under the ACT by unilaterally changing routes. Our new NE model under the ACT criterion shares similar properties with deterministic multi-class NE model, and can be solved by the traditional Frank–Wolfe algorithm. We also