Transportation Research Part B 93 (2016) 146–161

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

A method of integrating correlation structures for a generalized recursive route choice model

Tien Mai*

Department of Mathematics and Industrial Engineering, École Polytechnique de Montréal and CIRRELT, Canada

ARTICLE INFO

Article history:
Available online 30 July 2016

Keywords:
Recursive network MEV
Contraction mapping
Integrated network
Recursive cross-nested
Value iteration
Maximum likelihood estimation
Cross-validation

ABSTRACT

We propose a way to estimate a generalized recursive route choice model. The model generalizes other existing recursive models in the literature, i.e., (Fosgerau et al., 2013b; Mai et al., 2015c), while being more flexible since it allows the choice at each stage to be any member of the network multivariate extreme value (network MEV) model (Daly and Bierlaire, 2006). The estimation of the generalized model requires defining a contraction mapping and performing contraction iterations to solve the Bellman's equation. Given the fact that the contraction mapping is defined based on the choice probability generating functions (CPGF) (Fosgerau et al., 2013b) generated by the network MEV models, and these CPGFs are complicated, the generalized model becomes difficult to estimate. We deal with this challenge by proposing a novel method where the network of correlation structures and the structure parameters given by the network MEV models are integrated into the transport network. The approach allows to simplify the contraction mapping and to make the estimation practical on real data.

We apply the new method on real data by proposing a recursive cross-nested logit (RCNL) model, a member of the generalized model, where the choice model at each stage is a cross-nested logit. We report estimation results and a prediction study based on a real network. The results show that the RCNL model performs significantly better than the other recursive models in fit and prediction.

1. Introduction

Given a transport network with links and nodes, and given a origin-destination pair, the route choice problem deals with identifying which route that a traveler would take. Discrete choice models are generally used in the context. There are two main issues associated with the use of discrete choice models, namely, choice sets of paths are unknown to the analyst, and path utilities may be correlated. We consider a generalized recursive route choice model where the choice at each stage is a family of multivariate extreme value (MEV) models. The model has the advantages from the existing recursive models (Fosgerau et al., 2013a; Mai et al., 2015c), as it can be consistently estimated and is easy for prediction without sampling of choice sets. It is, moreover, flexible in the sense that it allows to capture the correlation at each choice stage by many convenient static discrete choice models, e.g., multinomial, nested logit (Ben-Akiva, 1973), cross-nested logit (Vovsha and Bekhor, 1998) or multi-level cross-nested models (Daly and Bierlaire, 2006). However, the generalized model is impractical to estimate if using the estimation methods proposed for the other recursive models. We therefore propose an innovative

* Corresponding author.
E-mail address: anh-tien.mai@polymtl.ca

http://dx.doi.org/10.1016/j.trb.2016.07.016

method that allows to integrate the correlation structure given by the model at each stage into the transport network and simplify the estimation.

The recursive logit (RL) model proposed by Fosgerau et al. (2013a) can be consistently estimated on real data without sampling any choice sets of paths. It is assumed that travelers choose states (nodes or links) in a sequential manner. At each state they maximize the sum of the random utility associated with a successor state (instantaneous utility) and the expected maximum utility from the state to the destination (also known as the value functions). The random terms of the instantaneous utilities are assumed to be independently and identically distributed (i.i.d.) extreme value type I and the RL model is equivalent to a multinomial logit (MNL) model over choice sets of all feasible path alternatives. The RL model hence inherits the independence of irrelevant alternatives (IIA) property which is undesirable in a route choice setting (for instance Mai et al., 2015d). Recently, Mai et al. (2015c) proposed the nested RL (NRL) model that relaxes the IIA property of the RL model by assuming that scale parameters are link specific. Both the RL and NRL models, however, assume that the choice at each stage are MNL, so the correlation between the utilities of successor states cannot be captured.

In this paper, we consider a generalization of these models where the choice model at each stage can be any member of the network MEV model (Daly and Bierlaire, 2006), e.g., the MNL, the nested logit (Ben-Akiva, 1973) or cross-nested logit (Vovsha and Bekhor, 1998). This model, called the recursive network MEV (RNMEV), is fully flexible in the sense that the cross-nested logit can approximate any additive random utility model (for instance Fosgerau et al., 2013b). However, the estimation of the generalized model leads to a complicated dynamic programming (DP) problem, which is cumbersome to solve. We explain this challenge in more detail in the following.

All the existing recursive route choice models are based on the dynamic discrete choice framework proposed by Rust (1987), where the estimation requires solving a Bellman's equation to obtain the value functions. Rust (1987) show that the value functions can be computed by defining a contraction mapping $V = \mathcal{T}_\beta(V)$, where $\mathcal{T}_\beta$ is a contraction mapping associated with parameters β, and V is the vector of the value functions. The fixed point solution, or the value functions, then can be obtained by using the method of successive approximations (or value iteration) based on $\mathcal{T}_\beta$. In practice, this method needs to be formulated as matrix operations in order to deal with problems with large number of states. In the RL and NRL models, due to the simple choice probability generating functions (CPGF) (Fosgerau et al., 2013b) given by the MNL model at the choice stages, the corresponding contraction mappings have simple forms (even linear form in the RL model). Consequently, the corresponding contraction iterations can be formulated easily as matrix operations, and the RL and NRL models can be estimated quickly using real networks. On the contrary, the CPGFs given by the network MEV model are complicated. They have closed forms but need to be computed recursively based on the network of correlation structure (see for instance Daly and Bierlaire, 2006). Hence, the contraction mapping given by the RNMEV model is complicated, and it is not straightforward to formulate the contraction iterations as matrix operations. This explains why the estimation of the RNMEV model is extremely expensive or even impractical if using directly the standard approaches proposed in Fosgerau et al. (2013a); Mai et al. (2015c) and Rust (1987). In this context, we note that the generalized model also relaxes the well-known CLOGIT assumption from the Rust's model (Rust, 1987), i.e., the assumption that the choice at each stage is MNL.

We propose an innovative method that allows us to simplify the contraction mapping given by the RNMEV and to quickly estimate the generalized model on real data. More precisely, we consider the networks of correlation structures given by the network MEV models (also known as MEV-networks) at the choice stages. We create a new artificial network by integrating the MEV-networks into the transport network and associate the states and arcs of the new network with new parameters and utilities based on those from the original network and the structure parameters of the network MEV models. We show similarities between the value functions given by the RNMEV model and those given by the NRL model on the integrated network. This method allows us to avoid recursively computing the CPGFs given by the network MEV models, and to use the contraction mapping from the NRL model to obtain the value functions of the RNMEV model. This approach, therefore, greatly simplifies the estimation of the RNMEV model.

We apply the new method on real data by proposing a recursive cross-nested logit (RCNL) model, a member of the RNMEV model, where the choice model at each stage is a cross-nested logit model (Vovsha and Bekhor, 1998). In the RCNL model, the variance-covariance matrix at each choice stage is no longer diagonal as in the RL and NRL models. This model therefore exhibits a more general correlation structure, compared to the other recursive models. We provide estimation and prediction results for a real network, and show that the RCNL model can be estimated in reasonable time while performing significantly better than the RL and NRL models in fit and prediction.

This paper makes two main contributions. First, we propose the method of integrating correlation structures to simplify the estimation of the RNMEV. Second, we propose the RCNL model that can flexibly capture the correlation structure at each choice stage, and we provide estimation and cross-validation results of the RCNL for a real network using real observations. Moreover, the estimation code for estimating the RCNL is available in an open source project and we share it freely upon request.

The paper is structured as follows. Section 2 introduces the RNMEV model and Section 3 presents the method of integrating correlation structure. Section 4 discusses in detail the maximum likelihood estimation. Section 5 presents the RCNL model. We provide the estimation results and a cross-validation study in Section 6, and finally, Section 8 concludes. The proofs of the theorems presented in this paper are provided in Appendixes A and B.