Transportation Research Part B 93 (2016) 181–206

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

Reliability-based stochastic transit assignment: Formulations and capacity paradox

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ARTICLE INFO

Article history:
Received 26 May 2015
Revised 15 April 2016
Accepted 21 June 2016
Available online 2 August 2016

Keywords:
Frequency-based transit assignment
Reliability-based user equilibrium
Variational inequality
Braess and capacity paradox

ABSTRACT

This study develops link-based and approach-based variational inequality (VI) formulations for the frequency-based transit assignment with supply uncertainty, where link flows and flow on each outgoing link from each node are decision variables, respectively. Both the mean and variance of travel cost, including the covariance of in-vehicle travel costs, are captured in both formulations. To address the covariance of in-vehicle travel costs between different links on the same transit line, an augmented route-section network representation is developed, allowing us to apply the dynamic programming method to compute the value of the mapping function of the VI. The approach-based formulation can be solved by an extragradient method that only requires mild assumptions for convergence. It is found that the number of links carrying flow and equilibrium cost can be underestimated if supply uncertainty is not considered.

The study also introduces and examines the capacity paradox, a phenomenon in which the network maximum throughput may be reduced after new transit lines are added to a transit network or after the frequency of an existing line is increased. It is found that the capacity paradox may or may not occur simultaneously with the Braess-like paradox, a phenomenon in which providing new transit lines to a network may deteriorate the network performance in terms of the total weighted sum of the mean and variance of travel cost of all of the passengers. The demand level and the degree of risk aversion of passengers are the key factors that determine the occurrence of the capacity paradox.

1. Introduction

1.1. Motivations and objectives

This research was motivated by two problems. First, due to various factors such as road incidents, signal breakdown, and weather conditions, the cost components of transit assignment problems are stochastic. Although some studies (e.g., Yang and Lam, 2006; Li et al., 2008, 2009b; Sumalee et al., 2011; Meng and Qu, 2013; Szeto et al., 2013; Fu et al., 2014) have developed models to capture the stochastic costs of transit assignment problems, these models have some of the following drawbacks: (1) their formulations require specific travel time distributions, which may not be validated in reality; (2) their

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http://dx.doi.org/10.1016/j.trb.2016.06.008
0191-2615/

algorithms require strong conditions (e.g., monotonicity) for convergence, which may not be satisfied by the cost function of the problem; and (3) their formulations (e.g., Szeto et al., 2013) are path-based. A traditional drawback of path-based models has been the large memory and storage requirements of path enumeration. Recently, efficient methods, such as event dominance (Florian, 1998, 2004) or equilibrated choice sets (Watling et al., 2015; Rasmussen et al., 2015), have been developed to overcome these issues and have been applied to commercial software packages (e.g., Emme). However, to the best of our knowledge, most existing path set generation methods assume deterministic travel costs, and the variance and covariance of travel costs are not addressed. These methods are sometimes heuristic and cannot be easily extended to capture the covariance terms in the variance path travel cost. Szeto et al. (2011, 2013) used a k-shortest path algorithm to generate paths as needed; however, from the perspective of efficiency, it is hard to determine a good choice for the value of k in advance and they did not demonstrate their solution methods using a large, realistic network.

Our second motivation is related to the use of transit assignment models to evaluate the effectiveness of network design strategies, such as adjustments to transit itineraries or service frequencies. Without the consideration of the demand and supply uncertainties, Szeto and Jiang (2014) revealed a Braess-like paradox, in which the system performance may deteriorate, in terms of the expected total system travel cost, after new transit lines are provided to a transit network or after the frequency of an existing transit line is increased. Other than the total system travel cost, which is a level of service measure, the network performance can also be measured by the network capacity, defined as the maximum throughput of a network, which determines whether the transit network can handle all of the demand. Thus, it is necessary to investigate how transit network design strategies affect network capacity, and whether there is a paradox from the perspective of transit network capacity; that is, the network capacity can be worse off after new transit lines are provided to a transit network or after the frequency of an existing transit line is increased, i.e., the capacity paradox.

The objectives of this study are as follows.

- To develop formulation approaches to model the transit assignment problem, in which stochasticities in the in-vehicle travel cost, dwell cost, and congestion cost caused by supply uncertainty are considered. To formulate the problem, these approaches only require the mean and variance of these cost components (and the covariance of in-vehicle travel costs) without specifying their distributions.
- To analyze the properties of the problem.
- To develop a convergent solution method under milder conditions that does not rely on path enumeration and generation techniques, and illustrate its performance to solve large transit networks.
- To introduce and examine the capacity paradox.

1.2. Literature review

Some of the early works in transit assignment or related areas, such as Lampkin and Saalmans (1967), Dial (1967), and Fearnside and Draper (1971), computed the shortest path and assigned passengers on it after accounting for waiting times at transit stops. However, these early models did not consider the route choice problem of passengers traveling between a pair of stops served by several competing, direct lines, where some of the routes may be overlapped.

Le Clercq (1972) did consider the route choice problem, but assumed that passengers consider all of the direct lines and board the first arriving bus. Chriqui (1974) and Chriqui and Robillard (1975) assumed that a passenger only considers a subset of these direct lines, so as to minimize his or her expected travel time. They solved the problem of selecting the optimal subset of direct lines analytically, a problem referred to as the common line problem.

The idea of a set of attractive lines has been generalized to the optimal strategy concept (Spiess, 1984; Spiess and Florian, 1989). Assuming that a passenger will use his or her individual optimal strategy in traveling, Spiess and Florian (1989) developed a linear programming model to tackle the common line problem; their proof demonstrated that their model's dual solutions satisfy the user equilibrium conditions. Subsequently, two modeling streams were derived from the abovementioned behavioral assumption using two different network representations: the hyperpath graph network representation (Nguyen and Pallottino, 1988; Wu et al., 1994; Cominetti and Correa, 2001; Cortés et al., 2013; Sun et al., 2013) and the route-section network representation (de Cea and Fernández, 1993; Lam et al., 1999, 2002; Li et al., 2009b; Szeto et al., 2011, 2013). Although both of these modeling approaches are based on the same behavioral assumption, they have different pros and cons. The merit of the hyperpath graph representation is that the optimal set of attractive lines can be easily determined, but at the cost of creating more boarding and alighting nodes. The route-section representation can reduce the number of links required to form the network when the number of common lines is large. Moreover, the route-section representation allows the development of a link-based formulation and the adoption of available algorithms to solve for solutions.

Other network representations were developed from the hyperpath or route-section network representations, including: (a) state augmented network (Lo et al., 2003, 2004; Lozano and Storchi, 2001); (b) space time network (Nguyen et al., 2001; Hamdouch and Lawphongpanich, 2008; Hamdouch et al., 2014); (c) diachronic network (Nuzzolo et al., 2001; Sumalee et al., 2009); and (d) star network (Tong and Wong, 1999; Zhang et al., 2010). In general, a more complicated network representation requires more memory storage and computation time; however, it captures more of the cost components considered by passengers, such as non-linear transit fare, transfer cost, and congestion cost.