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Testing for regularity and stochastic transitivity using the structural parameter of nested logit

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ABSTRACT

We introduce regularity and stochastic transitivity as necessary and well-behaved conditions respectively, for the consistency of discrete choice preferences with the Random Utility Model (RUM). For the specific case of a three-alternative nested logit (NL) model, we synthesise these conditions in the form of a simple two-part test, and reconcile this test with the conventional zero-one bounds on the structural ('log sum') parameter within this model, i.e. $0 < \theta \leq 1$, where θ denotes the structural parameter. We show that, whilst regularity supports the lower bound of zero, moderate and strong stochastic transitivity may, for some preference orderings, give rise to a lower bound greater than zero, i.e. impose a constraint $l \leq \theta$, where $l > 0$. On the other hand, we show that neither regularity nor stochastic transitivity constrain the upper bound at one. Therefore, if the conventional zero-one bounds are imposed in model estimation, preferences which violate regularity and/or stochastic transitivity may either go undetected (if the 'true' structural parameter is less than zero) and/or be unknowingly admitted (if the 'true' lower bound is greater than zero), and preferences which comply with regularity and stochastic transitivity may be excluded (if the 'true' upper bound is greater than one). Against this background, we show that imposition of the zero-one bounds may compromise model fit, inferences of willingness-to-pay, and forecasts of choice behaviour. Finally, we show that where the 'true' structural parameter is negative (thereby violating RUM – at least when choosing the 'best' alternative), positive starting values for the structural parameter in estimation may prevent the exposure of regularity and stochastic transitivity failures.

1. Introduction

As is well-established in microeconomic consumer theory, the fundamental preference axioms of completeness, transitivity and continuity – taken together – permit the representation of an individual's complete preference ordering by a continuous real-valued order-preserving function (Debreu, 1954). An important proposition follows from Debreu; the individual is conceptualised as making consumption choices *as if* to maximise utility. This proposition, which is the cornerstone of Neo-Classical consumer theory, has been the subject of considerable interest in the behavioural economics literature. A focus of this interest has been the design and implementation of experiments that seek to elicit empirical support for (or refutation of) the axioms of completeness, transitivity and continuity – as well as other related properties of choice behaviour.

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Emanating from this literature, several phenomena have been identified as giving rise to violations of the fundamental axioms and, by implication, violations of utility maximisation.

This paper is motivated by an interest in exploring analogies to the fundamental preference axioms, and their empirical verification, in the alternative domain of probabilistic discrete choice. The discrete choice context, where the individual chooses from a finite and exhaustive set of mutually-exclusive alternatives, creates difficulties for conventional Neo-Classical consumer theory. This is because the theory employs marginal concepts derived using calculus; application to discrete choice has been described as *'awkward'* (McFadden, 1981 p199), and worse still *'impossible'* (Ben-Akiva and Lerman, 1985 p44). In response to these difficulties, a bespoke version of consumer theory has evolved, centred upon the theoretical construct of the Random Utility Model (RUM)[1].

Drawing analogy with psychophysical models of judgement and choice (Fechner, 1859; Thurstone, 1927; Luce, 1959), RUM was conceived by Marschak (1960) and Block and Marschak (1960)[2] as a probabilistic representation of the Neo-Classical theory of choice. In common with the Neo-Classical theory, RUM is couched at the individual level, is based fundamentally on the notion that the individual acts *as if* to maximise utility, and (in the original 'distribution free' form of RUM proposed by B&M, at least) is entirely supported by the notion of ordinal utility. Contrasting with Neo-Classical theory, however, RUM appeals to the context of discrete choice consumption.

The present paper relates to three strands of extant literature, as follows.

1.1. Representation theorems for RUM

The literature on representation theorems has considered the necessity and sufficiency of conditions on probabilistic choice systems (PCS) giving rise to (cardinal) utility functions (Debreu, 1958; Davidson and Marschak, 1959) and RUM. Focussing here on representation theorems for RUM, Falmagne (1978) was first to show the necessity and sufficiency of the so-called 'B&M polynomials'[3]. Some years later (and apparently ignorant of Falmagne's paper until their attention was drawn to it in the course of peer review), Barberá and Pattanaik (1986) re-stated Falmagne's theorem in terms of rankings rather than utility scales, which allows closer correspondence with the concept of ordinal utility. More recently, Fiorini (2003) contributed an elegantly concise proof of Falmagne's theorem.

Mindful of its origins in the cognate discipline of psychophysics, it is interesting to observe that RUM has attracted interest from a multidisciplinary audience, spanning several core disciplines (especially economics, psychology and mathematics), as well as a raft of sectoral applications (including transport, health and the environment). McFadden (2005) presented a useful synthesis of representation theorems for RUM and, reflecting his parent discipline of economics, he characterised such theorems as addressing the 'problem of revealed stochastic preference'[4]. Within this synthesis, McFadden and Richter's (1970a unpublished) and 1970b papers, subsequently consolidated within their 1991 paper, covered similar ground to Falmagne (1978). Reflecting back some years later, Marley (1990) described the evolution of the literature on representation theorems for RUM, and offered specific observations concerning the links between the Falmagne and McFadden/Richter bodies of work.

A distinct but related strand of literature is that dealing with representation theorems for 'parametric' versions of RUM[5]. Motivated by an interest in its practical applicability, three independent parallel teams – namely Daly and Zachary (1976, subsequently published in 1978), Williams (1977) and McFadden (1978) – proposed alternative presentations of RUM, each formalised in terms of necessary and sufficient conditions on choice probabilities and/or random utilities giving rise to choice probabilities. In this context, and drawing similarities with McFadden's 'problem of revealed stochastic preference', the probabilistic content of RUM derives from the propensity for variability in behaviour across a population of individuals, as distinct from the intra-individual variability of a single individual in B&M. This change in emphasis, together with the extended theoretical apparatus, provided the stimulus for the adoption of RUM in mainstream econometric practice (see Section 1.3 to follow).

1.2. Empirical testing of theoretical properties of choice

Following from the theoretical developments outlined above, a second strand of literature has subjected the fundamental preference axioms – as well as a broader range of theoretical properties of choice – to empirical testing. In this context, the psychology and behavioural economics literatures would seem rather more developed than the discrete choice literature,

[1] One of the reviewers of this paper pointed out that the term 'Random Utility Model' (RUM) has sometimes been interpreted differently in different disciplines, and that a tighter and more contemporary terminology is 'choice probabilities induced by strict linear orders'; see Marley and Regenwetter's (2016) recent review of deterministic and probabilistic representations of choice, which distinguished between economic (i.e. parametric) and psychological (i.e. linear order) approaches to RUM. Since the terminology 'choice probabilities induced by strict linear orders' is not common parlance in transport, this paper will remain faithful to 'RUM', but the reviewer's point is worthy of mention.

[2] Henceforth, we will abbreviate Block and Marschak (1960) to 'B&M'.

[3] See Theorem 4 (p60) of Falmagne (1978).

[4] According to McFadden (2005), this problem poses the question: *'Are the distributions of choices observed for a population of individuals in a variety of choice situations consistent with rational choice theory, which postulates that individuals maximize preferences?'* (p245).

[5] In this regard, Regenwetter et al. (2010) distinguished between B&M's 'distribution free' RUM and the 'parametric' RUM that arises from (1), whilst Batley (2008) distinguished between 'ordinal' RUM and 'cardinal' RUM.