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A multiphase optimal control method for multi-train control and scheduling on railway lines

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ABSTRACT

We consider a combined train control and scheduling problem involving multiple trains in a railway line with a predetermined departure/arrival sequence of the trains at stations and meeting points along the line. The problem is formulated as a multiphase optimal control problem while incorporating complex train running conditions (including undulating track, variable speed restrictions, running resistances, speed-dependent maximum tractive/braking forces) and practical train operation constraints on departure/arrival/running/dwell times. Two case studies are conducted. The first case illustrates the control and scheduling problem of two trains in a small artificial network with three nodes, where one train follows and overtakes the other. The second case optimises the control and timetable of a single train in a subway line. The case studies demonstrate that the proposed framework can provide an effective approach in solving the combined train scheduling and control problem for reducing energy consumption in railway operations.

1. Introduction

Energy consumption in the railway industry is not only a concern for railway operators, but also attracts attentions from the academia. It is a consensus that a well-designed train control strategy could significantly reduce the energy consumption during the train run.

The train control problem is usually described as minimising the energy consumption of a train travelling from one station to the next within a given time period. The optimal control theory is a powerful tool for analysing such problem. In the pioneer work of Ichikawa (1968), the continuous train control problem under speed restriction was analysed as an optimal control problem with bounded state variables. Milroy (1980) suggested that, for short journeys, the optimal train control strategy consists of three control stages: maximum acceleration, coasting, and maximum braking. Asnis et al. (1985) and Howlett (1990) found that, for long journeys, an additional speedholding stage should be included, leading to an optimal sequence of acceleration, speedholding, coasting, and deceleration. Later research showed that these four distinct modes can be used to create optimal control strategies for very complex train operation problems involving variable track gradient (including steep climbs and steep descents), variable speed limit, complex train characteristics, and power regeneration (Albrecht et al., 2015b, 2015c; Howlett, 2000; Khmelnitsky, 2000; Liu and Golovitcher, 2003).

Different to the above-mentioned continuous train control problems, for some diesel locomotives, only discrete throttle settings are available. There, the speed may not be perfectly held for the speedholding operation. Under the discrete settings, Cheng and Howlett (1992, 1993) optimised the switching points of a prescribed number of control phases and showed that,

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in the simplest case of level track without speed limit, the speedholding can be approximated to any desired accuracy by a sequence of coast-power pairs. The discussion was further extended to the case of undulating tracks (Howlett, 1996; Howlett and Cheng, 1997), variable speed limits (Pudney and Howlett, 1994) and a combination of both (Cheng et al., 1999). The state-of-the-art review on both continuous and discrete train control problems can be found in Howlett et al. (2009) and Albrecht et al. (2015b, 2015c).

Efficient algorithms are essential for implementing optimal train control in real-life train operations. Numerical methods based on the Pontryagin's maximum principle has been well developed by Albrecht et al. (2015b, 2015c), Howlett et al. (2009), Khmelnitsky (2000) and Liu and Golovitcher (2003), and some of them have been implemented in the practical real-time driver advisory systems such as the *Energymiser*[1]. Genetic algorithms have been used to determine the optimal starting points and the lengths of coasting phases (Chang and Sim, 1997; Wong and Ho, 2004). Discretisation and approximation are often used to convert the original optimal train control formulation to the mathematical programming problems, such as dynamic programming method (Effati and Roohparvar, 2006; Franke et al., 2000; Ko et al., 2004; Vasak et al., 2009), linear programming method (Effati and Roohparvar, 2006; Wang et al., 2013, 2014) and pseudospectral method (Wang and Goverde, 2016; Wang et al., 2013, 2014). There are also numerical methods that don't belong to the above-mentioned types. For example, in Gu et al. (2014), the whole section between two stations is first divided into several subsections according to gradients and speed limits. The optimal control scheme on each of these subsections is derived and then used to obtain the control strategy for the complete journey through nonlinear programming. For the classical train control problems, the numerical methods based on the Pontryagin's maximum principle might be generally superior to other numerical methods mentioned above; however, for optimal train control in a general network, which is our focus in this paper, there is so far no efficient algorithms based on the Pontryagin's maximum principle, and in this case other numerical methods may be able to provide satisfying solutions.

In the classical train control problem, a single train is considered to run freely between two stations with prescribed journey times. However, this framework can face challenges in a complex and busy railway network. As it was argued by Albrecht et al. (2015b),

> "... the most pressing research challenges for the future in this area are to develop optimal control policies for trains travelling in the same direction on the same line in such a way that safe separation is maintained between trains. On busy rail networks, solution of the train separation problem relates to and depends on integrated scheduling and control policies to ensure that train movements are both energy-efficient and effectively coordinated."

The quote above raises two research gaps: the control of multiple trains running simultaneous on the same track, and the integration of train scheduling with optimal train control.

It happens in both fixed-block and moving-block systems that two or more trains can run sufficiently close to each other on the same track, and thus affect each other's operation. For example, when a train is expected to arrive at its frontal station ahead of schedule at full speed, it can slow down to save energy; however, the decision to slow down may affect the operation and thus the energy consumption of the train following it. In this case, the trajectories of both trains would be better optimised simultaneously. Research on this issue is still limited so far, as noted in Açıkbaş and Soÿlemez (2008), Albrecht et al. (2015a), Lu and Feng (2011), Miyatake and Ko (2010), Wang et al. (2014b), Yan et al. (2016) and Zhao et al. (2015). Specifically, Lu and Feng (2011) used genetic algorithm to optimise the control strategies of leading and following trains in a four-aspect fixed-block signalling system. Wang et al. (2014b) considered two trains traversing a single track following each other, under both fixed and moving block systems. They used the mixed integer linear programming approach to formulate the problems, and then solved the control strategies sequentially (the control of the leading train is solved first and then fixed when solving the control of the following train) or simultaneously. Albrecht et al. (2015a) provided theoretical analysis and solution algorithm for the train separation problem based on the optimal control theory, in which two trains were operated under the fixed-block system with specified starting and finishing times on level tracks without speed limit. Yan et al. (2016) developed an online distributed cooperative approach for optimising the control of multiple trains based on model predictive control and ant colony optimisation, where the trains were assumed to share their states and decisions with each other through radio.

Energy consumption is a big concern for the train operators as it directly affects the operators' profit margin. However, when it comes to train scheduling (over a large railway network), time efficiency is usually the first priority (Cacchiani and Toth, 2012; Cacchiani et al., 2014; Guo et al., 2016). There have been studies where energy consumption is incorporated in the scheduling process at an aggregated level. For example, in Medanic and Dorfman (2002) and Ghoseiri et al. (2004), the energy cost of a train running on a railway section is assumed to be a convex function of the average velocity of that train on that section.[2] If a more precise estimation on the energy consumption is expected, knowing the average speed is not enough. Instead, the detailed train running information at each time and location of the journey, including traction and braking forces, speed, and resistance, are required. However, these details are difficult to incorporate in the currently widely-used scheduling methods such as mathematical programming (Carey, 1994; Higgins et al., 1996) and discrete event simulation

[1] http://www.ttgtransportationtechnology.com/energymiser/.

[2] It was confirmed in Albrecht et al. (2016) that the energy consumption on level track is a strictly decreasing and strictly convex function of journey time. A new explicit formula was also provided in their paper for the rate of change of energy consumption with respect to journey time.