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A general stochastic process for day-to-day dynamic traffic assignment: Formulation, asymptotic behaviour, and stability analysis

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ABSTRACT

This paper presents a general modelling approach to day-to-day dynamic assignment to a congested network through discrete-time stochastic and deterministic process models including an explicit modelling of users' habit as a part of route choice behaviour, through an exponential smoothing filter, and of their memory of network conditions on past days, through a moving average or an exponentially smoothing filter. An asymptotic analysis of the mean process is carried out to provide a better insight. Results of such analyses are also used for deriving conditions, about values of the system parameters, assuring that the mean process is dissipative and/or converges to some kind of attractor. Numerical small examples are also provided in order to illustrate the theoretical results obtained.

1. Introduction

The development, since the 1970 s, of efficient computational methods for implementing network equilibrium models has arguably had one of the most significant impacts of academic research on transport planning practice. In many countries, such methods are an embedded element of procedures for cost-benefit analysis of proposed schemes, and are used widely for operational planning of traffic measures. With this class of approach now extended to consider multiple classes of users, within-day dynamic traffic interactions, unreliability and heterogeneity/mis-perception of users, their potential applicability is wider than it has ever been. Such facts are important to appreciate when proposing any approach that may be viewed as an *alternative* to the network equilibrium philosophy. Many large transport investments have been justified on the basis of equilibrium predictions, and so there is a political 'price' in practitioners moving to any "new" method. Academic researchers can help considerably in this process by better understanding the linkages between what might appear to be apparently diverse methods, and in particular by understanding the connection of any alternative approaches to network equilibrium. The objective, for example, could be to better understand the cases in which network equilibrium may be justified as an approximation to some real-world situation, and those cases in which it may potentially give misleading results. The present paper is motivated by exactly this desire to better understand the connections between approaches, and to understand where network equilibrium is a useful notion in this context. This includes the possibility, in some cases, that we calculate

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equilibrium in exactly the same way as we do at present, but the *meaning* or conceptualisation of the computed state is different, and suggests additional or alternative ways to utilise the computed state.

The focus of the present paper will be on what have become known as day-to-day dynamic models of route choice, focusing on (among other elements) how users adapt their route decisions over repeated trips. The term ‘day-to-day’ dynamic is useful to distinguish these approaches from ‘within-day dynamic’ models, these latter focusing on issues such as time-dependent OD demand rates, the spatial and temporal interactions of traffic flows, the influence on users’ time-dependent choice of route and possibly departure time, and the possibility for users to make *en route* diversions during a journey. In order to focus our discussion, we do not consider within-day dynamic issues in the present paper, though we note that there are several papers that consider the combination of day-to-day and within-day dynamics (Cascetta and Cantarella, 1991; Balijepalli and Watling, 2005; Liu et al, 2006; Friesz et al, 2011), and note that it is possible to transfer many of the arguments of the kind used here (admittedly at the price of far greater complexity) to the combined case. We note that the term ‘day-to-day dynamic’ is intended, therefore, to be indicative of the kind of process being considered, but it need not be that these models are representing a real, continuous sequence of complete days. In this respect it is good to have in mind the following suggestion of an ‘epoch-to-epoch dynamic’ model, where:

> *‘... epochs can have either a “chronological” interpretation as successive reference periods of similar characteristics (e.g. the a.m. peak period of successive working days) or they can be defined as “fictitious” moments in which users acquire awareness of path attributes and make their choices’. (Cascetta, 1989)*

There exist two clear classes of model of day-to-day dynamic route choice, namely Deterministic Processes (DPs) and Stochastic Processes (SPs). DPs are more naturally associated with traditional equilibrium models of transport systems, in the sense that point equilibria may emerge, under certain assumptions, as the convergent limits of such processes under some long-term steady conditions. DPs also allow transitions to be examined, especially when some ‘shock’ or designed change occurs (He & Liu, 2012). A recent review and synthesis of DP models in discrete and continuous time may be found in Cantarella & Wating, 2016; in this paper no comparison is carried out with DPs in continuous time and/or based on Wardrop approach to route choice behaviour (for a recent paper see Guo et al., 2015), since results obtained with these models can hardly transferred to the kind of models discussed in this paper.

SPs are more naturally associated with modelling the variability that is seen to occur in real-life systems, even under relatively stable operating conditions; they are thus able to represent both dynamic transitions and steady-state fluctuations. A review of SP models is provided in Watling & Cantarella (2013, 2015). The two types of approach draw on quite different mathematical disciplines, DPs emerging from non-linear dynamical system theory (typically interested in mappings over continuous state spaces), whereas SPs arising originally from the study of probability theory and Markov chains (over discrete state space).

Although some numerical evidence relating DPs and SPs exists (e.g. Cantarella and Cascetta, 1995; Watling, 1996), relatively little general, theoretical evidence exists concerning their relation for general traffic networks. The exceptions to this are the works of Davis and Nihan (1993) and Hazelton and Watling (2004), both of whom developed asymptotic approximation results for SPs, as demands and capacities grow in tandem. In the present paper we develop a ‘natural’ relation between DPs and SPs, which emerges from viewing DPs as a joint process in the statistical moments of the corresponding SP. This work is inspired by the general (asymptotic) theory mentioned above, and the series of two-link examples recently studied in Watling and Cantarella (2013). We shall here extend the work presented in Watling and Cantarella (2013) in several ways, particularly focusing on the development of the mean of a SP as a DP, as well as other results in literature.

The models presented and discussed in this paper extend our previous theoretical work on discrete-time stochastic and deterministic process models into a general modelling approach to day-to-day dynamic assignment so as to: (a) relate to general traffic networks (not just two-link networks); (b) include an inertia/habit effect modelled through an exponential smoothing filter; and (c) incorporate learning models with finite or infinite memory, bridging moving average and exponentially smoothed approaches.

The theoretical approximation of an SP model is first derived as a DP in the vector of flow means. Analysing the resulting DP, conditions are established to ensure uniqueness of the equilibrium, and to ensure its (local) asymptotic stability, conditions for the system being dissipative are also stated. Numerical examples are provided in order to motivate the work, to illustrate the theoretical results obtained, and to explore the generality of the asymptotic (large demand/large capacity) approximation, even in cases where demand and capacity are not “large” .

The paper is organised as follows. Section 2 presents basic notations and briefly reviews SUE models; then Section 3 discusses some simple but effective approaches to modelling dynamic learning and choice behaviour and analyses resulting Deterministic Process models. Section 4 describes the proposed SP model and some solution approaches as well as an asymptotic approximation to the mean of this process. Finally, in Sections 5 and 6 we discuss the main findings and identify several potential future research directions.

2. Basic notations, definitions and equations in SUE models

In this Section we will briefly review the basic notations and definitions adopted, as well as fixed-point models for stochastic user equilibrium assignment (Cantarella, 1997).