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Sensitivity analysis based approximation models for day-to-day link flow evolution process

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ABSTRACT

Compared with path-based day-to-day (DTD) traffic evolution models, link-based DTD traffic evolution models are easier to calibrate and validate. However, the inherent network loading sub-problem in link-based DTD models induces high computational burden which precludes their broad practical applicability. To address this challenge, this study proposes three approximation models for the DTD traffic flow evolution process based on the sensitivity analysis of the network loading sub-problem in a link-based DTD model. In particular, a first-order approximation (FOA) model is formulated based on the derivative of link flow solutions with respect to perturbations on network characteristics. To improve the approximation accuracy of the FOA model, a second-order approximation (SOA) model and a variable reduced approximation (VRA) model are developed. The applicability conditions of the proposed approximation models are derived. A small numerical example demonstrates that the FOA model performs well when perturbations are small and the approximation accuracy reduces as the scale of perturbations increases. The SOA and VRA models can improve the approximation accuracy of the FOA model, at the cost of computing the second-order derivative and the reference link flow pattern, respectively.

1. Introduction

Day-to-day (DTD) traffic evolution models describe how drivers adjust their daily path choice through learning based on past experience and projected information on future traffic conditions. In contrast to static traffic assignment models, which focus only on the final equilibrium state, DTD models are concerned more with the equilibration process which addresses whether and how the flow pattern evolves toward an equilibrium state. Since the early 1980s, a large number of DTD models have been proposed based on assumptions on the drivers' path choice adjustment behavior. According to Watling (1999), existing DTD models can be categorized into two classes: deterministic models (e.g., Smith, 1984; Zhang and Nagurney, 1996; Nagurney and Zhang, 1997; Friesz et al., 1994; Smith and Wisten, 1995; Huang and Lam, 2002; Yang and Liu, 2007; He et al., 2010; He and Liu, 2012; Smith and Mounce, 2011; Guo et al., 2015) and stochastic models (e.g., Cascetta, 1989; Davis and Nihan, 1993; Watling, 1999; Watling and Hazelton, 2003; Hazelton and Walting, 2004; Watling and Cantarella, 2013). Deterministic DTD models can be further divided into two classes: path-based models, (such as those summarized by Yang and Zhang, 2009) and link-based models (e.g., He et al., 2010; Smith and Mounce, 2011; Guo et al., 2015).

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The deterministic and stochastic dynamic processes have a close relationship. Davis and Nihan (1993) have shown that a deterministic dynamic process can be an appropriate approximation of the mean of the stochastic dynamic process when the driver population size increases. Cantarella and Cascetta (1995) further demonstrate this relationship in their study. Yang and Liu (2007) propose a modeling framework for developing mean deterministic dynamic processes to closely approximate a set of stochastic dynamic processes under the large population assumption.

Recently, several DTD models have been applied in developing analytical formulations for DTD traffic management, congestion control strategies, and prioritization of network recovery projects. For example, Yang et al. (2007) and Guo et al. (2013) developed DTD dynamic optimal tolling schemes that can lead the traffic flow to evolve toward system optimum. Cantarella (2013) studied the impacts of intelligent transportation systems on DTD traffic flow evolution process. Cantarella et al. (2013) applied DTD models to transit operation. Ye and Ukkusuri (2015) analyzed the optimal reconstruction sequence considering its impact on drivers' DTD path choice adjustment.

Although DTD models are theoretically appealing for modeling DTD dynamic traffic management and operational problems, they have not been widely applied in practice. A primary challenge faced by practitioners is the difficulty in model calibration and validation. Compared with a static traffic assignment model, a DTD model involves more parameters associated with the drivers' path choice adjustment behavior. Further, calibrating and validating a DTD model require traffic flow evolution data over a long time period rather than a single traffic flow pattern. In addition, a well-calibrated DTD model needs to be recalibrated before it can be used for another network or for the same network when demand or supply changes occur. A DTD model that is easy to calibrate and validate would increase its transferability in practice.

Among deterministic DTD models, link-based models are easier to calibrate and validate than path-based models. Calibrating path-based DTD models requires detailed information on path flows, which is practically difficult due to the path flow non-uniqueness problem inherent in path-based DTD models (He et al., 2010). In addition, tracing all vehicles' paths in the real world is a costly and technologically challenging process. By contrast, the calibration of link-based DTD models relies on link flow data that are easy to collect. Thereby, link-based DTD models are easier to deploy in current practice. For example, He and Liu (2012) calibrated and validated a link-based DTD model using link flow data collected before and after the collapse of the I-35W Mississippi River Bridge in Minneapolis, Minnesota.

Although the calibration and validation of link-based DTD models is relatively straightforward in practice, it is computationally more expensive than path-based DTD models. Due to the lack of path flow variables, link-based DTD models require a network loading sub-problem in their formulation to ensure the flow conservation. In particular, the link-based DTD model proposed by He et al. (2010) embeds an optimization sub-problem for network loading. The link-based splitting rate DTD model proposed by Smith and Mounce (2011) introduces an additional post-assignment link flow adjustment process to maintain flow conservation. The general link-based DTD model proposed by Guo et al. (2015) ensures the flow conservation through a projection operator that is equivalent to a minimization problem. As a sequence of traffic flow patterns are needed for model calibration and validation, the embedded network loading sub-problem would significantly increase the computational burden.

The high computational burden of solving link-based DTD models impedes their applicability to DTD traffic management and operational problems, where link-based DTD models can be used to describe the underlying traffic flow evolution process. To identify optimal solutions to DTD traffic management and operational problems, the DTD model needs to be solved multiple times to characterize the traffic evolution processes under different management and operational plans. Solving the underlying DTD model efficiently is crucial to quickly identifying the optimal solution from a large number of candidate plans, especially for DTD traffic management and operations responding to unexpected transportation network events.

To address the aforementioned challenges in applying link-based DTD models, there is a need to develop analytical models that can efficiently generate the traffic flow evolution process. Existing discrete-time link-based DTD models formulate the traffic flow evolution process as a difference equation, which can be seen as the first-order approximation of an ordinary differential equation for describing the continuous-time traffic evolution process. However, the embedded network loading in link-based DTD models destroys the simple structure of the difference equation, resulting in a computationally expensive formulation. If the network loading component can be replaced by an equality equation, the link-based DTD models can be simplified as a concise difference equation representation that is easier to calibrate and validate. Thereby, they can be efficiently adapted to model and solve DTD traffic management and operational problems.

This study seeks to develop three approximation models that are computationally efficient to characterize the DTD traffic flow evolution process. The proposed approximation models are constructed for a link-based DTD model. These models update the daily link flows without performing network loading. Thereby, they significantly reduce the computational burden in solving the link-based DTD model. In this paper, we will show that the proposed models can fit the original link-based DTD model under certain conditions. The capability for good approximation of the proposed models facilitates their applications in practice, as they can characterize the traffic flow evolution process similar to the link-based DTD model. In addition, the proposed approximation models can directly describe the link flow pattern on an arbitrary day without relying on an iterative process. Thereby, these models can be used to quickly assess the effectiveness of candidate traffic management and operational plans.

The study uses the link-based DTD model proposed by He et al. (2010) to develop approximation models due to its good analytical properties. First, it has a unique fixed point that is the attractor of convergent link flow evolution trajectories. Second, the nonlinear program of the network loading can be reformulated as the classical user equilibrium (UE) traffic assignment model, which can be approximated using sensitivity analysis methods.

The proposed approximation models are based on sensitivity analysis. They apply the sensitivity analysis for the UE model to obtain the derivatives of link flow patterns with respect to the perturbed parameters. Applying the first-order and second-order