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Day-to-day dynamical model incorporating an explicit description of individuals' information collection behaviour☆

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ABSTRACT

To forecast the performance of a congested transport system, it is necessary to model how transport system users adjust their day-to-day choice behaviour in an equilibration process. Although there are various ways of constructing a day-to-day dynamical model, the model should explicitly reflect the microscopic day-to-day adjustment behaviour of individual users to clarify which behavioural factors are incorporated in the model. From this point of view, existing day-to-day dynamical models seems not to contain sufficiently explicit mechanisms for the information collection behaviour. Several existing models implicitly incorporate user behaviour under imperfect information, but not in a way that can be explicitly related to the microscopic information acquisition processes of individual users. The present study proposes a continuous and deterministic day-to-day dynamical model that explicitly incorporates microscopic user behaviour about the information collection and can be handled easily in a mathematical manner. A microscopic model describing individuals' day-to-day adjustment process is first constructed. Then, introducing the mean-field approximation and assuming the large number of users, a macroscopic model that does not contain any disaggregated variable is derived as an ordinary differential equation. Convergence towards a user equilibrium solution is globally or locally guaranteed for utility functions associated with a potential function and those of monotonically decreasing regardless of the variety of information collection behaviour considered by the proposed model. On the other hand, results of numerical examples imply that the variety of information collection behaviour affects how a system behaves in a transient status of the dynamics, including how the system oscillates when it does not converge to an equilibrium point.

1. Introduction

To forecast the performance of a congested transport system under given supply and demand conditions, it is necessary to model how transport system users adjust their day-to-day choice behaviour in an equilibration process. The concept of user equilibrium (UE) has long been used to describe the final state of the equilibration process that would be realised in the real world. However, it is commonly known that the UE solution is insufficient for both theoretical and practical reasons. The theoretical reason is related to the properties of the UE solution, such as uniqueness and stability. These properties are

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not natural in traffic assignment problems with asymmetric cost functions (Watling, 1996). The issue is more prominent in within-day dynamic assignment problems (Iryo, 2013); for example, the uniqueness of the UE solution is not generally guaranteed (Iryo, 2011; Iryo, 2015). In addition, the existence of the unique UE solution does not always imply its stability (e.g. Iryo, 2008 for a simple departure time choice problem). The possible existence of multiple or unstable equilibrium solutions necessitates the modelling of day-to-day adjustment processes to identify a realistic equilibrium point. In addition to these theoretical motivations, there is a practical importance to analysing the day-to-day adjustment process to investigate traffic states in transient situations such as disaster recovery (He and Liu, 2012).

The concern about the day-to-day adjustment process in a transport system actually dates back to Beckmann et al. (1956), mentioning that '*the study of stability hinges ultimately on the question of how road users adjust themselves to changes – that is, how they adapt the extent of their travel by road and their choices of routes to varying traffic conditions*'. Works by Horowitz (1984), Smith (1984), and Cascetta (1989), are the pioneers proposing and analysing mathematical models describing the day-to-day adjustment process, which is now called 'day-to-day dynamics'. Then, this research area has emerged by Friesz et al. (1994); Cascetta and Cantarella (1993); Dupuis and Nagurney (1993); Cantarella and Cascetta (1995); Zhang and Nagurney (1996); Watling (1999), and subsequent studies. The comprehensive reviews and classifications of existing studies can be found in e.g. Watling (1999); Yang and Zhang (2009); and Watling and Cantarella (2013).

Day-to-day dynamical models naturally correlate with users' day-to-day adjustment behaviour. Hence, when proposing a model, stating assumptions on users' behaviour is important in order to clarify which factors are incorporated in the model. Indeed, the sentence by Beckmann et al. (1956) cited above is followed by '*This, however, is one of the big unknowns of road-user behavior, so that at the present stage only conjectures are possible*'. This note implies that users' day-to-day adjustment behaviour is not a matter of course like Wardrop's first principle (Wardrop, 1952), which is very simple and has been widely accepted as a criterion describing equilibrated transport networks.

Information collection behaviour should be one of the most important factors of users' day-to-day adjustment behaviour and consequently it must be properly assumed to propose the day-to-day dynamical model that can adequately describe a situation to be analysed. The primal reason why it is important is that it must directly affect users' day-to-day adjustment behaviour. The term of 'adjust' naturally implies that users seek an alternative whose utility is greater than the current one. If they cannot collect information of a certain alternative, they cannot know whether selecting it instead of another alternative is a better thing or not to adjust their choice behaviour. The other reason is that, in reality, users' information collection behaviour is largely affected by policies of information provisions. For example, consider a situation in which we attempt to construct a day-to-day dynamical model to evaluate performances of an advanced traveller information system (ATIS) to be installed in a city. To the evaluation, we need to consider at least two cases, i.e. one with ATIS and one without ATIS. Of course, users' information collection behaviour should be largely different between these cases owing to the difference of availability of information. In addition, it should be noted that ATIS is not a tool that automatically injects information on all the links and routes to all the drivers. In reality, the utilisation of ATIS is also subject to users' behaviour of information collection. For example, if the service of ATIS is charged, its price must affect the information collection behaviour of drivers. If the information acquisition of ATIS is based on probe vehicles, information provided by ATIS can be biased because route choices of probe vehicles are affected by information collected by them and consequently information on less used routes is less distributed. Regarding the first point, Chorus et al. (2006) reviews studies of information collection behaviour to evaluate advanced traveller information services (ATIS). Regarding the second point, Iryo et al. (2012) showed that the strong bias of information diffusion can occur in a non-recurrent travel behaviour model, in which people may be locked in an inferior alternative with lower utility owing to the lack of information.

That all users obtain information on past situations of all alternatives (such as routes) is perhaps the simplest assumption about information collection behaviour. This assumption was first explicitly stated by Horowitz (1984). Cascetta (1989) also incorporate the same assumption in the proposed learning mechanism. This assumption has been also incorporated in other studies (Cascetta and Cantarella, 1993; Cantarella and Cascetta, 1995; Watling, 1999; Watling and Hazelton, 2003; Watling and Cantarella, 2013; Xiao and Lo, 2015; Cantarella and Watling, 2016), mostly accompanied with a learning filter such as an exponential smoothing filter and a choice switching process. Owing to these accompanying mechanisms, this assumption may not only correspond to the extreme situation in which 'information on all the alternatives is automatically injected to all users' but also reflects more mild situations. Nevertheless, it should be pointed out that the above-mentioned assumption does not consider the possible biases of information owing to the information collection behaviour.

Effects made by the bias of information may be implicitly described by other existing models, but not in a way that can be explicitly related to the information collection behaviour of individual users. The proportional-switch adjustment process proposed by Smith (1984), also called Smith dynamics (Sandholm, 2010b), assumes that drivers swap from an alternative (route) to another route whose utility is *higher* (i.e. not necessarily highest) at a rate that is proportional to the difference of their utilities. This assumption implies that the model may implicitly consider incompleteness of information because the users can move to an alternative whose utility is not highest. However, in this definition, it is not sure what sort of information collection behaviour relates to the dynamics. Later, Dupuis and Nagurney (1993) proposed the projected dynamical system and Friesz et al. (1994) proposed a dynamical model incorporating incomplete information called the network tatonnement model. Similar to Smith dynamics, these two models were constructed with no explicit consideration of each individual's information collection behaviour. All of these models only include macroscopic state variables describing the aggregated behaviour of users (e.g. the number of travellers using each route) that does not include any information directly associated to each user. Their dynamics is described by ordinary differential equations (ODE) using a continuous