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Dynamic pricing in discrete time stochastic day-to-day route choice models

Tarun Rambha*, Stephen D. Boyles

Department of Civil, Architectural and Environmental Engineering, The University of Texas at Austin, United States

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ABSTRACT

The traffic assignment problem is primarily concerned with the study of user equilibrium and system optimum and it is often assumed that travelers are perfectly rational and have a complete knowledge of network conditions. However, from an empirical standpoint, when a large number of selfish users travel in a network, the chances of reaching an equilibrium are slim. User behavior in such settings can be modeled using probabilistic route choice models which define when and how travelers switch paths. This approach results in stochastic processes with steady state distributions containing multiple states in their support. In this paper, we propose an average cost Markov decision process model to reduce the expected total system travel time of the logit route choice model using dynamic pricing. Existing dynamic pricing methods in day-to-day network models are formulated in continuous time. However, the solutions from these methods cannot be used to set tolls on different days in the network. We hence study dynamic tolling in a discrete time setting in which the system manager collects tolls based on the state of the system on previous day(s). In order to make this framework practical, approximation schemes for handling a large number of users are developed. A simple example to illustrate the application of the exact and approximate methods is also presented.

1. Introduction

Urban transportation planning is traditionally carried out using a four-step process. The first three steps are used to estimate the number of travelers/users, their origin-destination (OD) pairs, and their mode of travel. The final step, also called *route choice* or *traffic assignment*, involves assigning travelers to different routes. This assignment procedure is done assuming that traffic networks are in a state of *user equilibrium* (UE) or *Nash equilibrium* (NE) due to selfish choices made by travelers (Nash, 1951; Wardrop, 1952). Many efficient algorithms exist for finding the UE solution to the traffic assignment problem (TAP) (Bar-Gera, 2002; Dial, 2006; Jayakrishnan et al., 1994; Larsson and Patriksson, 1992; Mitradjieva and Lindberg, 2013). Another state typically of interest is called the *system optimum* (SO) in which the sum total of travel time experienced by all travelers, also called the *total system travel time* (TSTT), is minimized.

In UE models, it is often assumed that travelers are rational and have a perfect knowledge of the network topology and its response to congestion. However, when a large number of travelers interact, the extent of reasoning required to arrive at an equilibrium solution is beyond one's human ability. Two alternate concepts which do not rely on these assumptions

* Corresponding author. Tel.: +1 5127612551.
E-mail address: tarun.1988@gmail.com (T. Rambha).

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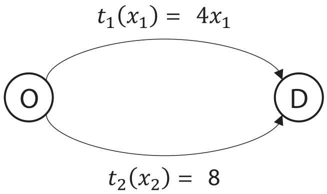

Fig. 1. Sub-optimality of marginal prices in a day-to-day setting.

exist in literature — *stochastic user equilibrium* (SUE) and *day-to-day dynamic models or Markovian traffic assignment models*. Both these approaches infuse uncertainty into travelers' choices and the uncertainty is assumed to result from randomness in users' perceived travel times. However, they differ from each other in a vital way. Stochastic user equilibrium models (Daganzo and Sheffi, 1977; Dial, 1971; Sheffi, 1985), which are formulated as fixed point problems, define equilibrium as a state in which users' perceived travel times are minimized.

On the other hand, day-to-day models (Cantarella and Cascetta, 1995; Cascetta, 1989; Friesz et al., 1994) are deterministic or stochastic dynamic processes in which states/feasible flows evolve over time. In discrete time models with stochastic dynamics, travelers select paths each day based on historical information of network travel times and a probabilistic route choice mechanism which induces transitions from one state to another. Under certain mild conditions, the stochastic process can be modeled as a Markov chain with a unique steady state distribution. Thus, although the system is never at rest, it attains an 'equilibrium' in the probability distribution of flow patterns.

Since paths are selected randomly on each day, the total system travel time is no longer deterministic but is a random variable. Using the steady state distribution of the stochastic process, one can compute the expected TSTT, which can be used as a metric for studying the extent of congestion in the network. An immediate question of interest is the following. Just as congestion pricing is used to achieve SO flows in traffic assignment, can a system manager do the same to reduce the expected TSTT? Traditional congestion pricing is based upon marginal prices (Pigou, 1920). Congestion pricing helps reduce the TSTT as the UE flow on the network with marginal tolls results in a SO flow pattern in the original network. However, in a day-to-day setting, marginal prices are of little relevance. In fact, in some cases, they can result in increased TSTT as illustrated by the following example.

Consider two travelers from O to D in the network shown in Fig. 1. Let the vector (x_1, x_2) denote the state of the system, where x_1 and x_2 denotes the number of travelers on the top and the bottom path, respectively. The above system has three states (2, 0), (0, 2), and (1, 1), which we will refer to as states 1, 2 and 3, respectively. It is easy to verify that state 1 is a NE and state 3 is SO. Suppose both travelers use the logit choice model to select paths on each day, in which the probability of choosing the top and bottom paths are $\frac{\exp(-t_1(x_1))}{\exp(-t_1(x_1))+\exp(-t_2(x_2))}$ and $\frac{\exp(-t_2(x_2))}{\exp(-t_2(x_2))+\exp(-t_2(x_2))}$, where $t_1(x_1)$ and $t_2(x_2)$ represents the travel times as a function of the previous day's flow. The stochastic process is Markovian and the steady state probabilities of observing states 1, 2, and 3 are 0.5654, 0.1414, and 0.2932 respectively. Thus the expected TSTT is 16(0.5654) + 16(0.1414) + 12(0.2932) = 14.8272. Now suppose we price the network using marginal tolls (4 units on the top link and no toll on the bottom one) and assume that both travelers now replace the travel time functions in the logit choice model with generalized costs (travel time + toll). The steady state distribution of the Markov chain for states 1, 2, and 3 is 0.467, 0.467, and 0.066 respectively and the expected TSTT is 15.736 which is higher than before.

Selecting the right tolls in a day-to-day setting would thus require us to estimate the steady state distribution for each admissible toll pattern and select one that minimizes the expected TSTT. However, one can do better than such static tolling schemes by dynamically varying tolls. While dynamic tolling has received some attention in literature, most existing research focuses primarily on continuous time models. These studies use control theory to determine the optimal time varying toll as the system state evolves with time according to some deterministic dynamic (Friesz et al., 2004; Xiao et al., 2014). However, continuous time formulations are not really 'day-to-day' models and their solutions cannot be used to dynamically price a network over different days. A major contribution of this paper is in addressing this gap by proposing a dynamic day-to-day pricing mechanism in a discrete time setting that computes the optimal link tolls to reduce the expected TSTT. We formulate this problem as an infinite horizon average cost Markov decision process (MDP) and seek stationary policies that inform us the tolls as a function of the state of the system. In other words, the system manager observes the state or flow pattern and sets tolls, which are then revealed to the travelers. Travelers pick paths the next day in a probabilistic manner depending on the current state and revealed tolls.

Tolls in real world transportation networks are largely levied on freeways and hence the path choice set for travelers may be assumed to be small. However, even in sparse networks, presence of a large number of travelers results in an exponential number of states. Therefore, as with most MDPs, we are faced with the *curse of dimensionality* that prevents us from using this model on practical networks. To address this, we also propose simple approximation techniques using state space aggregation to handle instances with large number of travelers and demonstrate its performance on a small test network. For most part, we will restrict our attention to a single OD pair and the logit choice model for route selection. Extensions to more general settings are conceptually straightforward.

The rest of this paper is organized as follows. In Section 2, we describe the two approaches (discrete and continuous) to model the evolution of traffic as a stochastic process. We also discuss existing literature on dynamic pricing. Section 3