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A least squares procedure for the evaluation of multiple generalized stress intensity factors at 2D multimaterial corners by BEM

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ABSTRACT

Detailed characterization of linear elastic stress states at corners and crack tips requires knowledge of the stress singularity orders, the characteristic angular functions and the generalized stress intensity factors (GSIF). Typically a high accuracy is found in the literature for the evaluation of the stress singularity orders and characteristic angular functions (numerically computed from analytical expressions in most cases). Nevertheless, GSIF values, evaluated by means of a numerical model using FEM or BEM and usually by postprocessing the results, are often reported with a lower level of confidence. A robust procedure is presented in this work for the evaluation of the GSIF at multimaterial corners. The procedure is based on a simple least squares technique involving stresses and/or displacements, computed by BEM, at the neighborhood of the corner tip. A careful verification of the robustness and accuracy of the procedure using a few benchmark problems in the literature has been carried out. Applications of the procedure developed to the evaluation of GSIFs appearing at corners in metal-composite adhesive lap joints are presented.

1. Introduction

Problems having abrupt changes in geometry and/or material properties at some singular points solved under the assumptions of linear elasticity present unbounded stresses (referred to as stress singularities), see [1–8]. Neighborhood of such a point is usually referred to as corner, multimaterial corner if several materials meet at this point or also cross points if the singular point is located inside the domain. If the conditions for small scale yielding apply, the obtained singular elastic solution at a certain distance to the corner tip is representative of the real stress state. In this case, the linear elastic solution using a polar coordinate system (r,θ) centered at the corner tip, see Fig. 1, admits, with the exception of some degenerate cases, a representation for the stresses σ_{ij} and displacements u_i by the following asymptotic series expansion, see [9–12] for a rigorous mathematical justification, for $r \to 0_+$:

$$\left.\begin{aligned} \sigma_{ij}(r,\theta) &\cong \sum_{m=1}^{M} \frac{K_m}{r^{1-\lambda_m}} f_{ij}^m(\theta) \\ u_i(r,\theta) &\cong \sum_{m=1}^{M} K_m r^{\lambda_m} g_i^m(\theta) \end{aligned}\right\}, \quad (i,j = r,\theta,z) \tag{1}$$

in which λ_m are the singularity exponents, $f_{ij}^m(\theta)$ and $g_i^m(\theta)$ are, respectively, the characteristic angular functions for stresses and displacements, and K_m, the weights of the terms, are the so-called Generalized Stress Intensity Factors (GSIF). Thus, the set of K_m $(m=1, \ldots, M)$ defines the local elastic state at a corner. Notice that each term in (1) represents a solution of the governing partial differential equations in the corner domain. We will refer to each term as a mode similar to that used in the case of cracks. It is assumed that λ_m are naturally ordered fulfilling $\mathrm{Re}\lambda_m \leq \mathrm{Re}\lambda_{m+1}$. Logarithmic terms have not been considered in this work for the sake of brevity; see Sinclair [13] for further information. However, the procedure for the evaluation of GSIFs presented here may be easily generalized to include these logarithmic terms.

When a λ is a complex number, as in the case of interface cracks, $\lambda = \lambda_R + i\lambda_I$ (where λ_R and λ_I are real numbers), the associated GSIF is also a complex number $K = K_R + iK_I$ (where K_R and K_I are real numbers). In such a case, two terms can be included in (1). In the representation of the stresses, one term would be equal to $K_R \mathrm{Re}[r^{\lambda-1} f_{ij}(\theta)]$ and the other would be $K_I \mathrm{Im}[r^{\lambda-1} f_{ij}(\theta)]$, while in the representation of the displacements, one term would be equal to $K_R \mathrm{Re}[r^{\lambda} g_i(\theta)]$ and the other $K_I \mathrm{Im}[r^{\lambda} g_i(\theta)]$.

The characteristic angular functions $f_{ij}^m(\theta)$ and $g_i^m(\theta)$ together with the singularity exponents λ_m depend only on the local geometry, local material properties and the type of local boundary conditions. The GSIFs, K_m, in addition depend on the global geometry, material properties and prescribed boundary conditions, their values being proportional to the magnitude of boundary conditions for linear elastic solutions.

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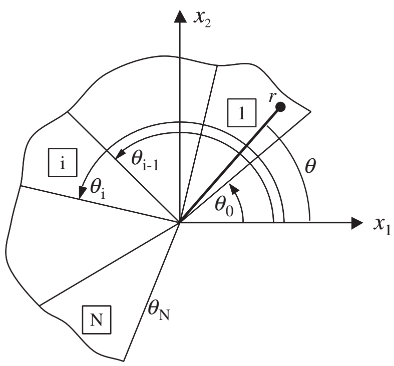

Fig. 1. Multimaterial corner.

Table 1
Classification of procedures for GSIF evaluation.

Procedure	Local techniques	Global techniques
Evaluation of GSIFs using postprocessing of numerical (FEM, BEM) results	Based, e.g., on least squares fitting [14–17]	Based, e.g., on conservative integrals [18–20]
Incorporation of singularity shape functions in the problem discretization	Quarter point elements [21–23] and other singularity elements [24]	Functions in the whole domain ([25] and [58]

The rigid body motions are included in (1) for $\lambda_m=0$ (translations) and $\lambda_m=1$ (rotations) with the appropriate definition of g_i^m and the corresponding $f_{ij}^m=0$. Note that $\mathrm{Re}\lambda_m>0$ for the other modes in (1).

The terms in (1) with $\mathrm{Re}\delta_m>0$, where $\delta_m=1-\lambda_m$, thus $0<\mathrm{Re}\lambda_m<1$, are called singular, giving rise to unbounded stresses as $r\to 0_+$. In this case, δ_m are referred to as the stress singularity orders.

Failure predictions at these corner-points admit several approaches, most of them being based either on allowable values of these GSIFs or on allowable values of field variables evaluated by means of the local stress or strain fields governed by the GSIFs. The evaluation of these GSIFs is then of crucial importance. For the evaluation of the GSIFs, the global geometry and far field loading must be considered. Thus, in general, numerical models by means of Finite Element Method (FEM) or Boundary Element Method (BEM) or experimental tests (e.g. using photoelasticity) have to be used. Techniques for the evaluation of GSIFs can be roughly divided into four basic groups, according to the local/global character of the procedure and to the necessity, or otherwise, of postprocessing of the FEM or BEM analysis results.

Local techniques are sensitive to the accuracy of the numerical solution values for stresses and/or displacements close to the corner tip, while global techniques, working also, or only, with the elastic solution far from the corner tip, typically by making use of conservative integrals, are thus less sensitive to the solution accuracy at the corner tip. Regarding the second criterion for the classification, those techniques using postprocessing of basic field variables do not need to be incorporated into the numerical codes (FEM, BEM), but do not typically have as good accuracy as the methods which directly incorporate the singularity shape functions into the discretization, usually requiring a modification of the numerical code.

Some references belonging to these groups are included in Table 1, which do not aim to be an exhaustive review. Further information can be found in Helsing and Jonsson [25], Sinclair [6,7] and Paggi and Carpinteri [8]. Particular examples of GSIF evaluation involving materials having non-isotropic constitutive laws can be found in Quaresimin and Ricotta [26], Nomura et al. [27] using the H-integral and including thermal stresses, or the interaction integral by Cisilino and Ortiz [28].

Typically, high accuracy can be found in the literature regarding the evaluation of singularity exponents λ_m and characteristic functions $f_{ij}^m(\theta)$ and $g_i^m(\theta)$ in (1), see Wieghardt [1], Williams [2] and Vasilopoulos [3] for single isotropic corners, Dempsey and Sinclair [4,5] for multimaterial isotropic corners, Mantič et al. [29] for single orthotropic corners, Pageau et al. [30] for single anisotropic corners, Mantič et al. [31] for multimaterial anisotropic antiplane corners, Ting [32], Barroso et al. [33], Hwu et al. [34] and Yin [35] for multimaterial anisotropic corners, which can be computed by finding the roots of an analytical function. However, the accuracy in the evaluation of GSIFs is substantially worse as numerical models of the global elastic problem and postprocessing of the results are needed. Reliable and accurate results for benchmark problems which could be used as reference values in new evaluation methods are needed. Some interesting comments regarding the validity of the published numerical results in the literature can be found in Helsing and Jonsson [36].

The approach proposed and explored in this paper is aimed at being an accurate and easy-to-use procedure for the evaluation of GSIFs. It can be located in the above classification in the first row (see Table 1), with no need to modify the FEM or BEM code applied, and between the two columns, as both near and far field data of the analysis results can be used. It should be noted that no special need for an accurate solution from the FEM or BEM analysis step in the neighborhood of the corner tip is required, and it has a certain robustness with the use of the far-field data (far from the corner tip).

The paper is divided into five main sections. Section 2 deals with the description of the basic features of the Boundary Element code (BEM) applied, París and Cañas [37] and Graciani et al. [38], used for the numerical models in this work. Section 3 describes the postprocessing procedure for the evaluation of GSIFs, which is based on a least squares method in terms of the displacements and/or stresses using the results obtained along the boundary edges of the corner and also the common edges between material wedges in the case of a multimaterial corner. The proposed postprocessing procedure for evaluation of GSIFs K_m ($m=1, \ldots, M$) is particularly well suited for numerical solutions arising from BEM models as the displacements and stress vector are the basic field variables used in the procedure. Nevertheless, no limitation appears in using the numerical solutions obtained by FEM models. Difficulties with the possible ill-conditioning of the resulting linear system are also analyzed in this section. Section 4 is devoted to the evaluation of the implemented procedure by means of the well-known benchmark problems from the literature, performing parametric analyses to check the accuracy and robustness of the procedure. Section 5 applies the procedure for the evaluation of GSIFs in configurations of multimaterial corners appearing in adhesively bonded double-lap joints between aluminum and carbon fiber laminates. A final section summarizes the main features of the work.

2. Multidomain BEM code

The present study involves the numerical analysis of 2D plane elasticity problems including one or multiple materials, with isotropic or orthotropic behaviors, in which singular stresses are present.

Two-dimensional BEM is employed in its traditional collocation formulation using continuous linear elements to mesh the boundaries of the problem in the so-called displacement Boundary Integral Equation (BIE).