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An EWMA monitoring scheme with a single auxiliary variable for industrial processes

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ABSTRACT

When using control charts to monitor manufacturing processes, Shewhart control chart is known to be useful for detecting transient shifts, while the EWMA and CUSUM charts are useful for detecting persistent shifts. The efficiency of EWMA chart in monitoring location parameter can be improved by using an auxiliary variable that is closely related to the variable of interest. In this paper, an EWMA-type scheme using ratio estimator is developed to further increase the effectiveness of the classical EWMA chart in monitoring the location parameter. The proposed procedure outperforms the classical EWMA and even the mixed EWMA-CUSUM chart, especially when there is a strong positive relationship between the variable of interest and the auxiliary variable. Finally, a real data set is used to show the implementation procedures of the proposed chart.

1. Introduction

Statistical process control (SPC) is a data-driven statistical method for monitoring and controlling a process in manufacturing industry. The Shewhart control chart (Shewhart, 1924) is an example of the memoryless-type control chart. The two most commonly used memory type control charts are the exponentially weighted moving average (EWMA) control chart and the cumulative sum (CUSUM) control chart, proposed by Page (1954) and Roberts (1959), respectively. The memory control charts utilize previous information with present information to yield a better result for detecting small to moderate shifts, unlike Shewhart-type charts that use only the current information. Research works related to the Shewhart-type charts and CUSUM-type charts can be found in Hawkins and Olwell (1998), Mukherjee and Sen (2015), Li, Mukherjee, Su, and Xie (2016), Chong, Mukherjee, and Khoo (2017), Lombard, Hawkins, and Potgieter (2017), and Sanusi, Riaz, Abbas, and Abujiya (2017). Different EWMA-type schemes have also been proposed in the literature. Noorossana, Fathizadan, and Nayebpour (2016) investigated the joint effect of non-normality and parameter estimation on EWMA chart. Also, Tamirat and Wang (2016) introduced an acceptance sampling plan scheme based on an EWMA statistic. In the case of unknown parameters, EWMA median control chart with estimated parameters was introduced to monitor the location parameter of a normal process (Castagliola, Maravelakis, & Figueiredo, 2016). Also, Zhou, Shu, and Jiang (2016) suggested a one-sided EWMA scheme with varying sample sizes for monitoring rare events. Furthermore, EWMA control chart found early applications in economics (Muth, 1960) and in inventory control and forecasting (Dushman, Lafferty, & Brown, 1962). For more works on the improvement of EWMA chart, interested readers can see Liu, Xie, Goh, and Chan (2007), Sheu, Tai, Hsieh, and Lin (2009), Teh, Khoo, and Wu (2011), Xie, Xie, and Goh (2011), Nishimura, Matsuura, and Suzuki (2015), and Zwetsloot, Schoonhoven, and Does (2016).

The traditional EWMA chart monitors the process mean (say $\overline{Y}$) of a process distribution. The $\overline{Y}$ is computed using the famous simple random sampling (SRS) approach. However, in the presence of an auxiliary variable (X) that is closely related to the study variable (Y), $\overline{Y}$ can be estimated more efficiently. Consequently, Cochran (1940) used the advantage of auxiliary information to propose a ratio mean estimator ($\overline{y}_r$) for estimating the population mean of Y. Choudhury and Singh (2012) noted that $\overline{y}_r$ is most effective when there is a positive linear relationship (which passes through the origin) between Y and X and the mean square error (MSE) of Y is proportional to X. Murthy (1964) suggested the use of $\overline{y}_r$ when $\rho_{YX} C_Y / C_X > 0.5$, where ρ_{YX}, C_Y, and C_X are, respectively, the correlation coefficient between Y and X, the coefficient of variation of the study variable (Y), and the coefficient of variation of X. Also, Adebola, Adegoke, and Sanusi (2015) introduced an efficient estimator, with cum-dual ratio estimator as intercept, for estimating the population mean.

Riaz (2008a) popularized the idea of using an auxiliary variable at

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the stage of estimating the plotting statistic of a monitoring chart. He presented a Shewhart-type scheme based on a regression-type estimator for detecting shifts in process variability and also showed the superiority of the scheme over the traditional Shewhart control chart. Also, a Shewhart-type control chart was suggested by Riaz (2008b) for monitoring process location. The chart is based on the regression mean estimator and it was shown that it outperforms the Shewhart's X - bar chart and the cause-selecting charts. The work was later extended to the EWMA set-up for detecting small to moderate shifts in the process mean (Abbas, Riaz, & Does, 2014). It was revealed that the chart outperforms other existing univariate and bivariate charts Also, some efficient estimators with an auxiliary variable are used to improve the performance of the combined Shewhart-CUSUM chart for detecting both small and large shifts in the location parameter of a process (Sanusi, Riaz, & Abbas, 2017).

In this article, an auxiliary variable is introduced at the estimation stage to monitor the location parameter of a process distribution. The proposed chart, denoted as MrEWMA, is an EWMA-type control chart based on the ratio mean estimator. This motivation further enhances the sensitivity of the control chart, especially in detecting shifts of small magnitudes. The average run length (ARL) approach is used to evaluate the performance of the chart. Also, other performance measures such as the standard deviation of the run length (SDRL), the extra quadratic loss (EQL), the relative average run length (RARL), and the performance comparison index (PCI) are considered. The management perspective of the proposed scheme is also briefly discussed.

One of the three activities for the successful execution of an efficient management of a process is quality control. This requirement ensures that products are up to standard through continuous improvement. The proposed MrEWMA scheme would further help to continosly improve the performance of a product, which will lead to a long-term reward for industries. Also, the early detection of shifts would avoid mass inspection in controlling quality, since a good quality is achieved by preventing defective items, instead of inspecting the items for bad products. Moreover, industry with a modern method of improving products quality, and can demonstrate process capability and control, has an edge over other competitors. These are in agreement with the Deming philosophy in improving management strategies (Montgomery, 2009).

The rest of this article is arranged as follows: The statistical preliminaries of the proposed scheme; the structural framework of the classical EWMA chart including its plotting statistic, control limits, and ARL; and the design of the proposed chart are presented in Section 2. The performance evaluations and the major findings of the proposed MrEWMA chart are provided in Section 3. The comparison of the MrEWMA control chart with its existing counterparts is given in Section 4. A real-life illustrative example is given in Section 5. Finally, the summary and conclusion of the findings are provided in Section 6.

2. The proposed monitoring scheme

The motivation of this work is to enhance the sensitivity of EWMA control chart in detecting shifts in the location parameter of a control process. This is achieved by introducing an auxiliary variable, in the form of a ratio estimator, to the charting scheme. In the next subsections, the mathematical preliminaries of the proposed scheme are explained, followed by a brief description of the classical EWMA, and then, the construction of the proposed MrEWMA chart.

2.1. Statistical preliminaries

Let X represents an auxiliary variable which is positively correlated with the study variable (Y) of a control process. The extra information provided by the auxiliary variable can increase the efficiency in estimating the population mean ($\overline{Y}$). One of the ways of integrating the extra information is to use the ratio mean estimator defined as:

$$\overline{y}_r = (\overline{y}/\overline{x})\overline{X}, \tag{2.1}$$

The estimator is biased, that is, $E(\overline{y}_r) = \overline{Y} + B(\overline{y}_r)$, where $E(\overline{y}_r)$ is the expectation of the ratio mean estimator $(\overline{y}_r)$ and $B(\overline{y}_r)$ is the bias of $\overline{y}_r$. The $B(\overline{y}_r)$ is defined as:

$$B(\overline{y}_r) = \overline{Y}(C_X^2 - \rho_{YX} C_Y C_X)/n, \tag{2.2}$$

For the Case-U, $B(\overline{y}_r)$ can be estimated as:

$$\hat{B}(\overline{y}_r) = \overline{y}(c_X^2 - r_{YX} c_Y c_X)/n, \tag{2.3}$$

where $C_X = \sigma_X/\overline{X}$, $C_X = \sigma_Y/\overline{Y}$, $c_X = s_X/\overline{x}$, and $c_Y = s_Y/\overline{y}$.

Furthermore, the mean squared error of $\overline{y}_r$ is given as

$$MSE(\overline{y}_r) = \overline{Y}^2(C_Y^2 + C_X^2 - 2\rho_{YX} C_Y C_X)/n, \tag{2.4}$$

which can be estimated as (Singh & Mangat, 1996)

$$mse(\overline{y}_r) = \overline{y}^2(c_y^2 + c_x^2 - 2r_{yx} c_y c_x)/n. \tag{2.5}$$

Note that sampling with replacement and approximation up to the first order are assumed. Also, the choice of ρ_{YX} that gives an efficient estimation is determined by the condition $\rho_{YX} > C_X/2C_Y$ (Singh & Mangat, 1996). More often, $C_X \approx C_Y$, which implies that $C_X/C_Y \approx 1$. Consequently, we have $\rho_{YX} > 0.5$. As a result, in the presence of an auxiliary variable, it is more efficient to apply the ratio mean estimator instead of the usual SRS mean estimator if $\rho_{YX} > 0.5$ (Murthy, 1964). Murthy (1964) noted that in the case of large samples and when both Y and X are both positive or negative, the product estimator, the usual mean estimator, and the ratio estimator should be used when $-1 \leqslant \rho_{YX} < -C_X/2C_Y$, $-C_X/2C_Y \leqslant \rho_{YX} \leqslant C_X/2C_Y$, and $C_X/2C_Y < \rho_{YX} \leqslant 1$, respectively. If either Y or X is negative, the conditions for the product and ratio estimators will be reversed. In Fig. 1, the broken horizontal line is the standard error (SE) of the usual SRS mean, while the curve is the SE of the ratio estimator which varies with different ρ_{YX} values. The two lines meet at a point $C_X/2C_Y$. It is shown that the SE of the ratio estimator is less than the SE of the usual SRS mean when $C_X/2C_Y < \rho_{YX} \leqslant 1$. In this research, it is assumed that both Y and X are greater than zero. Hence, only positive values of ρ_{YX} ($\rho_{YX} = 0.25$, 0.50, 0.75, and 0.95) are considered. Furthermore, the usual SRS estimator may be better than the ratio estimator when near proportionality between Y and X does not exist (Cochran, 1940). This happens when the regression line of Y on X passes through a point on Y-axis that is not close to the origin. Before constructing the proposed control chart, the classical EWMA will be introduced briefly.

2.2. The classical EWMA chart

The classical EWMA control chart is a memory-type chart that detects persistent shifts in a control process. It is frequently used for monitoring single observations, though it can also be used for monitoring process mean of rational subgroups of size $n > 1$ (Montgomery, 2009). Consequently, the procedures for constructing an EWMA chart

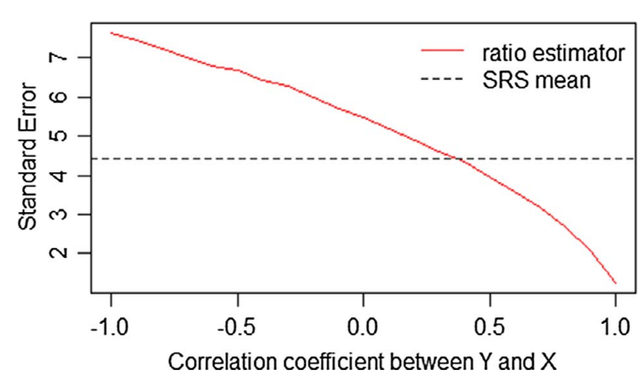

Fig. 1. Estimated SE of the ratio estimator vs. SE of the usual SRS mean estimator.