Computers & Industrial Engineering 113 (2017) 195–205

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

The newsvendor model with non-zero reference point based on cumulative prospect theory

Shou-Rong Li, Ren-Qian Zhang *, Wei-Guo Fang

School of Economics and Management, Beihang University, Beijing 100191, China

ARTICLE INFO

Article history:
Received 18 February 2017
Received in revised form 20 July 2017
Accepted 31 August 2017
Available online 5 September 2017

Keywords:
Newsvendor
Prospect theory
Cumulative prospect theory

ABSTRACT

This paper formulates the newsvendor model with a non-zero reference point based on cumulative prospect theory (CPT-based newsvendor model), evaluating the prospect by a piecewise-linear value function (*Model PL*). We prove the concavity of the objective function, and therefore, the model is solved by the first-order optimality condition. As a comparison, we further present the newsvendor model based on a piecewise-exponential value function (*Model PE*), where the utility curvatures are considered.

The results show that for a low-profit item, only *Model PE* can explain the "*pull to center*" effect that was found by Schweitzer and Cachon (2000), if the reference point is high enough. However, for a high-profit item, both models successfully predict the newsvendor's behavior if the newsvendor conceives a non-zero reference point. Thus, prospect theory (PT) should not be excluded as a potential explanation for the "*pull to center*" effect of the newsvendor's decision.

1. Introduction

Plenty of evidence indicates that newsvendors' decisions have significant consequences to firms (Carlson & O'Keefe, 1969; Petruzzi & Dada, 1999; Bendoly, Donohue, & Schultz, 2006; Zhang, Zhang, Zhou, Saigal, & Wang, 2014; Shu, Wu, Ni, & Chu, 2015; Abdel-Aal & Selim, 2017). In reality, newsvendors' order quantities often deviate from those that maximize the expected profit, i.e., the assumption of risk-neutral newsvendors is violated (Fisher & Raman, 1996; Shu et al., 2015; Abdel-Aal & Selim, 2017). Schweitzer and Cachon (2000) conducted two laboratory experiments to investigate newsvendors' *decision bias*. They reported that the average order quantity in the experiment is lower than the optimal order quantity of the risk-neutral newsvendor model in a high-profit condition, but higher than that in a low-profit condition. That is, the newsvendor prefers an order that deviates from the optimal risk-neutral order quantity and is biased in the direction of the mean demand; this is called the "*pull to center*" effect. In subsequent studies, experimental evidence also confirmed that the newsvendors' order quantities are biased to the mean demand (Benzion, Cohen, Peled, & Shavit, 2008; Bolton & Katok, 2008; Bostian, Holt, & Smith, 2008; Bolton, Ockenfels, & Thonemann, 2010). Schweitzer and Cachon (2000) believed that the experimental results are consistent with the explanations of anchoring and insufficient adjustment and a desire to reduce ex-post inventory error. Meanwhile, they ruled out the explanations of risk-neutral, risk-averse and risk-seeking preferences, as well as loss aversion, waste aversion, stock-out aversion, underestimated opportunity costs, and the prospect theory (PT).

PT was proposed by Kahneman and Tversky (1979), assuming that whether the outcome is coded as gain or loss is relative to a *reference point* rather than the final wealth. PT also assumes that the decision maker is loss-averse by adopting a value function. The value function is concave for gains, convex for losses, and is generally steeper for losses than for gains. Moreover, PT introduces a nonlinear transformation function depicting the overweight of small probabilities and the underweight of moderate and high probabilities, which helps to describe the fourfold pattern of risk attitudes and has a major effect on choice behavior. The original PT permits violations of stochastic dominance but is limited to a maximum of two non-zero outcomes (Wu & Gonzalez, 1996). Tversky and Kahneman (1992) proposed cumulative prospect theory (CPT) to handle more complex problems with multiple outcomes.

Regarding the newsvendor bias, after the work of Schweitzer and Cachon (2000), Nagarajan and Shechter (2013) also theoretically confirmed that "prospect theory cannot explain people's ordering decisions". However, their conclusions might be questionable because they overlooked all possible non-zero reference points. In this paper, we will reinvestigate the explanation for the "*pull to center*" effect based on CPT, using the same setup of

* Corresponding author.
E-mail address: zhangrenqian@buaa.edu.cn (R.-Q. Zhang).

http://dx.doi.org/10.1016/j.cie.2017.08.039
0360-8352/

Schweitzer and Cachon (2000). We propose two newsvendor models based on CPT: the first one is formulated by using a piecewise-linear value function (*Model PL*), mainly considering loss aversion depending on the reference point. The second one is based on a piecewise-exponential value function (*Model PE*), that simultaneously considers the loss aversion effect, risk aversion in positive prospect and risk seeking in negative prospect. The two models are used to predict the ordering behavior of newsvendors in Schweitzer and Cachon (2000).

The results show that CPT can largely explain the data of *Experiment 2* in Schweitzer and Cachon (2000) by introducing a non-zero reference point: (1) in a low-profit regime, *Model PL* cannot yield the order quantity of the "*pull to center*" effect because of the conservative decision caused by risk aversion and the loss aversion effect; (2) however, *Model PL* and *Model PE* can predict the "*pull to center*" effect in a high-profit regime, which can attribute to risk aversion and the loss aversion effect as well as the principle of "*diminishing sensitivity*"; (3) when considering risk seeking by the utility curvature under negative income in a low-profit regime with a high reference point, *Model PE* will appropriately predict the newsvendor's ordering behavior; (4) the weighting function of probabilities of CPT pulls the order quantity to the median demand.

We organize the paper as follows. Section 2 reviews the literature related to our work. Section 3 formally states the problem studied in this paper. Sections 4 and 5 explain the details of the two models and their solutions. Section 6 presents the computational results and discussions. Section 7 draws the conclusion.

2. Related literature

The study of behavioral operations management has thrived in recent years since the presentation of PT by Kahneman and Tversky (1979). PT resolves the paradox and alleviates some limitations surrounding expected utility theory (EUT). It has been widely used to predict individual preference under uncertainty (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992). Based on PT, Wang and Webster (2009) considered a zero reference point in a newsvendor model, asserting that the reference point will not affect the decision. However, the reference-dependent utility function of PT has successfully explained many systematic deviations of actual decisions from the risk-neutral decision (Shalev, 2000), and extensive evidences in recent years argue that behavior is affected by the reference point (Apesteguia & Ballester, 2009).

Reference point evolves naturally during the choice selection process (Kőszegi & Rabin, 2006; Herweg, 2013). Gavirneni and Xia (2009) found that the subjects did not anchor to the same information in all five settings. During the experiments of Schweitzer and Cachon (2000), the subjects could solicit the information regarding the profit distribution and the break-even sales level, or could view an updated table of results containing their past order quantities, realizations of demand, profit, and cumulative profit after each round. Therefore, it is reasonable to assume that the newsvendors are susceptible to their expectations.

Zhao and Geng (2015) considered a reference point based on the cumulative prospect function in Nagarajan and Shechter (2013), but they did not provide analytical analysis and just gave several numerical examples to show the possible power of PT in predicting the preferences of newsvendor's decisions. In this paper, we will investigate what will happen if non-zero reference points are introduced into the CPT-based newsvendor model, and reinvestigate the explanation for the "*pull to center*" effect based on CPT.

Gains above a psychological reference point and losses below a psychological reference point lead to different attitudes and have a significant influence on decisions. A risk-averse newsvendor tends to order less than the optimal risk-neutral order quantity while a risk-seeking newsvendor orders more (Eeckhoudt, Gollier, & Schlesinger, 1995). In CPT, risk attitudes are not only determined by utility curvature (sensitivity towards outcomes), but also by subjective probability weighting (sensitivity towards probabilities) and the loss aversion effect (Booij & Van De Kuilen, 2009). Most of the previous studies considered utility curvature or the loss aversion effect of CPT while Nagarajan and Shechter (2013) focused on the weighting of probabilities.

One closely related work to ours is Uppari and Hasija (2014), where the authors addressed the same issue under a similar setting and obtained a similar conclusion. However, the reference point in their model is fixed on the profit related to the mean demand, while ours changes between the lowest and highest possible profits. Besides, they did not obtain the optimal order quantity given a specific set of parameters. We presented the concavity of *Model PL* and therefore the solution can be easily worked out.

As a reference, we have also summarized the previous work involving PT/CPT in Table 1, showing their contributions in investigating inventory decision bias or the "*pull to center*" effect in the newsvendor context.

3. Theoretical foundation and model specification

3.1. The continuous form of CPT

To handle the newsvendor problem, we need the continuous form of CPT, which is a generalization from the case of multiple distinct outcomes to that of continuous outcomes (Tversky & Kahneman, 1992).

Suppose an individual makes a choice under risk with a continuous prospect $P = (\pi, p)$, where outcome π of prospect P is continuous with a probability density function (PDF) $p = f(\pi)$ and a cumulative distribution function (CDF) $F(\pi)$. According to CPT, the overall utility of a prospect is evaluated based on three functions: the decision weight w^+ for gains, the decision weight w^- for losses, and the value function $\upsilon(\pi)$. Let $\bar{F}(\pi) = 1 - F(\pi)$, the decision weight $w(f(\pi))$ and the valuation of the prospect $V(P)$ in CPT is defined by Eqs. (1) and (2), respectively.

$$w(f(\pi)) = \begin{cases} w^- = d[w(F(\pi))], & \text{if } \pi < 0 \\ w^+ = d[-w(\bar{F}(\pi))], & \text{if } \pi \geqslant 0 \end{cases}, \tag{1}$$

$$V(P) = \int_{\pi=-\infty}^{0} \upsilon(\pi) d[w(F(\pi))] + \int_{\pi=0}^{\infty} \upsilon(\pi) d[-w(\bar{F}(\pi))], \tag{2}$$

where the probability weighting functions w^- and w^+ are strictly increasing, satisfying $w^-(0) = w^+(0) = 0$, and $w^-(1) = w^+(1) = 1$.

A commonly used weighting function $w(p)$ is defined in Eq. (3) (see Prelec (1998)).

$$w(p) = e^{-(-\ln p)^\beta} \quad \text{and} \quad 0 < \beta < 1, \tag{3}$$

where e is the base of the natural logarithm. It is noted that $p_e = 1/e$ is the unique inflection point of the inverse-S-shaped weighting function in Eq. (3) for any $\beta \in (0, 1)$. $w(p) = e^{-(-\ln p)^\beta}$ is concave below and convex above the p_e point at which the function gets its minimum slope.

The magnitude of underweight (overweight) gets serious for smaller β. Wu and Gonzalez (1996) reported a pooled estimate of 0.74, with a standard error of 0.14. Nagarajan and Shechter (2013) set β equal to 0.60, 0.74, and 0.88. In this paper, we also consider the three values for β.