Computers & Industrial Engineering 113 (2017) 269–281

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Monitoring multivariate process variability via eigenvalues

Jinyu Fan[a], Lianjie Shu[a,*], Honghao Zhao[b], Hangfai Yeung[a]

[a] Faculty of Business Administration, University of Macau, China
[b] School of Business, Macau University of Science and Technology, China

ARTICLE INFO

Keywords:
Multivariate statistical process control
Variability
Sparsity
Covariance matrix
Eigenvalues

ABSTRACT

Various methods have been proposed to monitor changes in a process covariance matrix. In view that a covariance matrix can be fully defined by its eigenvalues and eigenvectors, this paper suggests monitoring the covariance matrix based on eigenvalues as another alternative. Although there are some recent discussions about the use of eigenvalues for hypothesis testing in multivariate analysis, the use of them for monitoring covariance matrix changes has been less studied in multivariate quality control. The simulation results show that the proposed method performs especially well under simultaneous shifts in both variance and correlation elements and competitively under shifts in variance or correlation elements only, compared to the existing approaches. This demonstrates a good property of the proposed method being able to provide a robust detection performance under a wide variety of scenarios. A real example is also provided to illustrate the implementation of the proposed method.

1. Introduction

Product quality has been the key driver in manufacturing/remanufacturing decision making (Diallo, Venkatadri, Khatab, & Bhakthavatchalam, 2017). To achieve continuous quality improvement, statistical quality control methods have been widely used. In modern manufacturing industries, it is very common that multiple quality characteristics are measured due to rapid development in information retrieval and storage technologies. Aimed at making use of the correlation structure among variables, multivariate statistical process control (MSPC) techniques have been widely discussed in the past two decades (Stoumbos, Reynolds, Ryan, & Woodall, 2000; Woodall, 2001; Woodall & Montgomery, 1999). Beginning with the seminal work by Hotelling (1947, chap. ii), there is a great deal of work devoted to the development of multivariate control charts for monitoring both the process mean vector or/and covariance matrix. Bersimis, Psarakis, and Panaretos (2007) provided an extensive review for the existing multivariate control charts. Some recent contributions to the development of MSPC techniques include Alfaro, Alfaro, Gamez, and Garcia (2009), Zou and Qiu (2009), Du, Lv, and Xi (2012), Costa and Machado (2013), Wu et al. (2015), Das, Zhou, Chen, and Horst (2016), Qi, Wang, Zi, and Li (2016), Yu and Chen (2016), Li, Pu, Tsung, and Xiang (2017).

Like the monitoring of process mean vector, it is also important to monitor process covariance matrix, which describes the variance of each variable and the covariance between any two variables. A number of methods have been proposed for monitoring covariance matrices. The very early work can be traced back to Montgomery and Wadsworth (1972). They first proposed the generalized variance method by charting the determinant of the sample covariance matrix. A major drawback of it is the poor detection ability in the case when some elements of the variability increase while others decrease. Another alternative is based on the generalized likelihood ratio (GLR) test. A sample of research in this category includes Alt (1984), Alt and Bedewi (1986), Alt and Smith (1988), Levinson, Holmes, and Mergen (2002), and Vargas and Lagos (2007).

Note that the GLR test proposed by Alt (1984) is essentially a Shewhart-type control chart. It is sensitive for detecting large transient changes in the covariance matrix but less sensitive for detecting smaller persistent shifts. In order to improve the performance in detection small shifts, some cumulative methods are further suggested. For example, Healy (1987) proposed a cumulative SUM (CUSUM) control chart to detect the change of covariance matrix. Similarly, some authors suggested the multivariate exponentially weighted moving average (MEWMA) control charts based on the likelihood ratio test for monitoring covariance matrices. See, for example, Yeh, Lin, Zhou, and Venkataramani (2003), Yeh, Huwang, and Wu (2005, 2004), Huwang, Yeh, and Chien-Wei (2007) and Hawkins and Maboudou-Tchao (2008). Some other variations include Memar and Niaki (2009) and Variyath and Vattathoor (2014).

However, most of the above control charts do not take into account process knowledge gleaned from engineering and operations. When the

* Corresponding author.
E-mail address: ljshu@umac.mo (L. Shu).

http://dx.doi.org/10.1016/j.cie.2017.09.025
Received 21 April 2017; Received in revised form 15 September 2017; Accepted 16 September 2017
Available online 20 September 2017
0360-8352/© 2017 Elsevier Ltd. All rights reserved.

process covariance matrix changes, it is typically the case in practice that only a small subset of elements would be affected (Li, Wang, & Yeh, 2013). The change in the covariance matrix will be reflected in a few variance and/or covariance elements. This is so-called "sparsity" characteristic. It would be more efficient for a control chart to monitor these few changed elements, compared to the monitoring of all the elements in the covariance matrix. In order to obtain a sparse estimate of the covariance matrix, the penalized likelihood ratio (PLR) approach is often employed. Different penalty terms can be added to the GLR for different purposes. For example, Yeh, Li, and Wang (2012) proposed the LASSO based MEWMA control chart for monitoring covariance matrix under the case with individual observations. The penalty is based on the L_1 norm of the difference between the precision matrix (or the inverse of the covariance matrix) and the in-control covariance matrix. Later, Li et al. (2013) further developed a PLR-type control chart for more general cases with subgroup size larger than or equal to the number of variables. The penalty term is based on the L_1 norm of the precision matrix. Maboudou-Tchao and Agboto (2013) applied the graphical LASSO estimator of the covariance matrix under the case with few observations than variables. Similarly, Maboudou-Tchao and Diawara (2013) discussed the graphical LASSO estimator of the covariance matrix under the case with individual observations.

The above PLR-type control charts have been shown to perform well in many cases. However, their appealing performance heavily depends on the choice of the tuning parameter to achieve different level of sparsity, when estimate the dispersion matrix or covariance matrix. The tuning parameter has significant effect on the control chart performance at different shift magnitudes. Unfortunately, the tuning parameter is not easy to specify. To accommodate such problem, Shen, Tsung, and Zou (2014) recently proposed a MaxNorm chart to monitor changes in a covariance matrix. The basic idea is to first calculate the deviation of the estimated covariance matrix from the target matrix, and then to transform the resulting matrix into a deviation vector, and finally to assess if the process covariance matrix changes based on the L_2 norm and the L_∞ norm of the deviation vector.

The objective of this paper is to suggest monitoring the covariance matrix based on eigenvalues as another feasibility. This is motivated by the fact that a covariance matrix can be uniformly displayed by its eigenvalues and eigenvectors. The eigenvalues describe the key characteristics of the covariance matrix such as determinant and trace. Moreover, the likelihood ratio is also closely related to eigenvalues. Therefore, in addition to the use of deviation vector/matrix to assess changes in the covariance matrix, it is also feasible to determine changes in the covariance matrix based on changes in the eigenvalues. The basic idea of the proposed method is to make a combined use of the L_2 norm and the L_∞ norm of the estimated eigenvalues to track changes in the covariance matrix. There are some discussions about the use of eigenvalues for testing sphericity, i.e., if the population covariance matrix is proportional to the identity matrix. See, for example, Marčenko and Pastur (1967), Johnstone (2001, 2009) and Nadler (2011). However, the use of eigenvalues for monitoring the covariance matrix has been less studied in MSPC. One exception is Noor and Djauhari (2011) in which they suggested the use of the largest eigenvalue of the sample covariance matrix for monitoring multivariate process variability. Note that the proposed method differs from the method of Noor and Djauhari (2011) in two aspects. First, in addition to the use of the largest eigenvalue, the proposed method also makes use of the L_2 norm of the estimated eigenvalues for monitoring. Second, Noor and Djauhari (2011) used the sample covariance to estimate the covariance matrix, and thus their method is not applicable to the case with individual observations. Instead, the proposed method is more flexible, which can work in both the case with individual observations and a subset of observations.

The rest of the paper is organized as follows. In Section 2, we briefly review some existing control charts. In Section 3, the proposed control chart is presented. In Section 4, extensive simulations were performed to compare the performance between the proposed control chart and other existing charts numerically. In Section 5, a real example is provided to illustrate the use of the proposed chart. Finally, some concluding remarks are discussed in Section 6.

2. The existing control charts

Assume that the process observations are independently and identically distributed with multivariate normal $N_p(\boldsymbol{\mu},\boldsymbol{\Sigma})$, where $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are the process mean and covariance matrix, respectively. In this study, we focus on Phase-II monitoring of changes in the covariance matrix, assuming that the mean vector $\boldsymbol{\mu}$ is known and fixed. Moreover, assume that the in-control covariance matrix, $\boldsymbol{\Sigma} = \boldsymbol{\Sigma}_{IC}$, is known or can be accurately estimated from Phase I data. Denote $\boldsymbol{\Sigma} = \boldsymbol{\Sigma}_{OC}$ as the out-of-control covariance matrix. The objective is to detect if the process variability changes from $\boldsymbol{\Sigma}_{IC}$ as soon as possible. Without loss of generality, $(\boldsymbol{\mu},\boldsymbol{\Sigma}_{IC})$ can be simplified to $(\mathbf{0},\mathbf{I}_p)$ by standardizing data with $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}_{IC}$. These assumptions are consistent with those made in Shen et al. (2014).

Let $\boldsymbol{x}_{t,1},\ldots,\boldsymbol{x}_{t,n}$ be the set of n observations with dimension $p \times 1$ collected in the subgroup at time t. When the sample covariance matrix is used to estimate the underlying process variance matrix, a rational subgroup of observations is often required. To relax this limitation, Hawkins and Maboudou-Tchao (2008) further considered a more general estimate based on the multivariate exponentially weighted moving covariance (MEWMC), which can be applied to both the case with individual observations and the case with a subgroup of observations. The MEWMC is defined as

$$\boldsymbol{\Sigma}_t = (1-\lambda)\boldsymbol{\Sigma}_{t-1} + \lambda\mathbf{S}_t, \quad t = 1,2,\ldots, \tag{1}$$

where $\mathbf{S}_t$ is a $p \times p$ sample covariance matrix given by

$$\mathbf{S}_t = \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{x}_{t,i}\boldsymbol{x}_{t,i}^T = \frac{1}{n}\mathbf{X}_t\mathbf{X}_t^T, \tag{2}$$

and $\mathbf{X}_t = (\boldsymbol{x}_{t,1},\ldots,\boldsymbol{x}_{t,n})$ is a $p \times n$ matrix. The initial value of $\boldsymbol{\Sigma}_t$ is set to $\boldsymbol{\Sigma}_0 = \mathbf{I}_p$. The MEWMC estimate in Eq. (1) works when both $n = 1$ and $n > 1$. Based on the covariance matrix estimate $\boldsymbol{\Sigma}_t$, the log-likelihood ratio can be obtained as

$$LR_t = \log\left\{\frac{f_1(\boldsymbol{x}_{t,1},\cdots,\boldsymbol{x}_{t,n}|\boldsymbol{\Sigma}_t)}{f_0(\boldsymbol{x}_{t,1},\cdots,\boldsymbol{x}_{t,n}|\boldsymbol{\Sigma}_{IC})}\right\} = -\frac{n}{2}(\log|\boldsymbol{\Sigma}_t|-\log|\boldsymbol{\Sigma}_{IC}|) -\frac{n}{2}[tr(\mathbf{I}_p)-tr(\boldsymbol{\Sigma}_{IC}^{-1}\boldsymbol{\Sigma}_t)]$$

where $tr(\cdot)$ is the trace operator, and $|\cdot|$ represents the determinant. Omitting unneeded constants, the above log-likelihood ratio for comparing $\boldsymbol{\Sigma}_t$ with $\boldsymbol{\Sigma}_{IC} = \mathbf{I}_p$ reduces to

$$LR_t \propto tr(\boldsymbol{\Sigma}_t)-\log|\boldsymbol{\Sigma}_t|-p. \tag{3}$$

Hawkins and Maboudou-Tchao (2008) proposed a control chart, denoted as the HMT chart for simplicity, to monitor the process variability based on the simplified likelihood ratio, i.e.,

$$T_{t,HMT} = tr(\boldsymbol{\Sigma}_t)-\log|\boldsymbol{\Sigma}_t|-p, \tag{4}$$

which triggers an out-of-control signal when $T_{t,HMT}$ exceeds a threshold.

The HMT chart is shown to be powerful in detecting shifts that occur in the majority elements of the covariance matrix. However, shifts may occur only in a few of the variance/covariance elements in practice. To take the advantage of the sparsity of shifts, some PLR-type control charts have been proposed. For example, Li et al. (2013) developed one

Download English Version:

https://daneshyari.com/en/article/5127435

Download Persian Version:

https://daneshyari.com/article/5127435

Daneshyari.com