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Periodical preventive maintenance contract for leased equipment with random failure penalties

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ABSTRACT

Production decisions for leased equipment often depend on many market factors. In this study, we take the penalty of changing market environment into account for the expected total cost model of preventive maintenance (PM) to obtain an optimization PM strategy. The lessor in the proposed model will incur a penalty for overdue minimal repair time. The penalty is considered as a function of expected revenue during the period of minimal repair, and the Black-Scholes equation is used to model the penalty function for establishing the expected total cost model. An optimal PM strategy is obtained to minimize the expected total cost for lessor through an analytical optimal procedure. Numerical examples for Weibull lifetime equipment are used to illustrate the applications of the proposed method under different scenarios.

1. Introduction

With complex and varied demands for production equipment, numerous producers would like to lease equipment rather than buy equipment, see Desai and Purohit (1998), Kleiman (2001), and Nisbet and Ward (2001). Since equipment failure causes damages for producers, maintenance strategies are frequently contained in leasing contracts. Generally, maintenance strategies comprise two types, the preventive maintenance (PM) and corrective maintenance (CM). PM is an intended strategy to slow deterioration and reduce the failure risk of equipment. CM is a strategy of fixing and recovering failed equipment.

Most often, CM is adopted through using "minimal repair" method. Minimal repair describes the situation where the equipment returns normal operations following maintenance. Nakagawa (1981) mentioned that the failure rate after maintenance would return that at the time of equipment failure. Nakagawa and Kowada (1983), Sheu (1991) and Yeh and Lo (2001) also considered minimal repair as a factor for the CM strategy. Liu, Xu, Xie, and Kuo (2014) proposed a dynamic PM policy for the system with continuous degrading components. Wang et al. (2016) proposed a period preventive policy for the modular multilevel converter. Lee and Cha (2016) proposed a generalized version of nonhomogeneous Poisson process model for periodic PM policies for a deteriorating repairable system. Regarding a PM, we consider an incomplete PM operation to reduce the probability of equipment failure under cost considerations. But the equipment may not return to its original condition following a PM. Many authors had studied the incomplete PM strategies, some of them are Jayabalan and Chaudhuri (1992), Nakagawa (1979), Nakagawa (1988), Pham and Wang (1996), Rangan and Grace (1989), Sheu and Griffith (1992), Lu, Chen, Liu, and Zhao (2012), Chen (2012), Dhouib, Gharbi, and Aziza (2012), and Zhao, Nakagawa, and Qian (2012), and Driessen, Peng, and Houtum (2017).

Numerous studies investigated the optimal PM strategies for leased equipment, for example, Barlow and Proschan (1965), McCall (1965), Dekker (1996), Dekker and Scarf (1998), Pham and Wang (1996), Pieskalla and Voelker (1976), Sherif and Smith (1981), Valdez-Flores and Feldman (1989), Chang and Lo (2011), Yeh, Kao, and Chang (2009), Schutz and Rezg (2013), Zhao, Wu, Li, and Xi (2016), Xia, Xi, Pan, Fang, and Gebraeel (2017). Yeh and Chen (2006) investigated optimal PM strategy for the lifetime of leasing equipment following the Weibull distribution. Jaturonnatee, Murthy, and Boondiskulchok (2006) extended the work of Yeh and Chen (2006) to obtain optimal PM strategies under general conditions. However, almost existing studies focused on obtaining optimal PM strategies based on production and maintenance costs and assumed constant revenue. The above existing models cannot fully take account for the effects caused by changing market environment. In this paper, the factors of changing market environment, including the fluctuations in demand and prices of products, are considered to find optimal PM strategies.

Recently, real option method has been widely applied to managerial decision making for assessing the uncertainty of business environment,

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see for examples Merton (1998) and Li (2007). Some studies have employed real option method to analyze production and quality control issues, see Lint (1992), Nembhard, Shi, and Park (2000), Bengtsson and Olhager (2002), Nembhard, Shi, and Aktan (2002). It is noted that Nembhard et al. (2002) considered uncertain market variables and designed a model for a manufacturing process; they assumed that assignable causes could hurt the quality of products and used Black-Scholes equation, binomial and multinomial lattices to evaluate real options. Nembhard et al. (2002) conducted a case study to evaluate their proposed methods through using numerical methods. To the best of our knowledge, this study is the pioneer to use Black-Scholes equation approach to investigate optimal periodical PM strategies for leased equipments. The concepts of using Black-Scholes equation to model penalty function, based on the overdue minimal repair time, for leased equipment is introduced, and the optimal PM strategy to minimize an expected total cost function is also obtained.

The remainder of this paper is organized as follows. Section 2 defines the total cost model and optimal PM strategy. The implementation of using some specified process for penalty, which is a function of overdue minimal repair time, to obtain an optimal PM strategy is studied in Section 3. Numerical examples for Weibull lifetime equipment are presented in Section 4 to illustrate the applications of the proposed method under different scenarios. Some conclusions are drawn in Section 5.

2. The total cost model

We use the following notation:

The parameters of the lifetime distribution of leased equipment:
α: the location parameter of Weibull distribution
β: the shape parameter of Weibull distribution

The parameters of the price, demand, and revenue process:
$D(t)$: the product demand within time $t S(t)$: the price of product at time $t R(t)$: the revenue process
r: the non-risk interest rate

The parameters of the PM strategy:
T: leased period
$N(t)$: the number of equipment failures in the period $[0,T] \lambda(t)$: the intensity function of $N(t) \tau_n$: the time between two PMs; if there are n PMs in the policy, then $\tau_n = T/(n+1)$
x_i: the degree of i th PM, $0 \leqslant x_i < i\tau_n - \sum_{k=0}^{i-1} x_k$, $x_0 = 0 x^{(i)}$: the cumulative age reduction in the first i PMs
$\lambda_n(t|\widetilde{x})$: the failure rate function under an n PMs policy with the degree vector $\widetilde{x} = (x_1,...,x_n) \Lambda_n(t|\widetilde{x})$: the expected number of failures within $[0,t) c_r$: the minimal repair cost
$c_m(x_i)$: the PM cost
Y: minimal repair time
δ: the repair time limit
a: the random penalty coefficient
K: the upper bound of the random penalty
$L(S(t),D(t))$: the random penalty at time t
$EC_M(n,\widetilde{x},T)$: the expected total cost function under the PM strategy $(n,\widetilde{x},T)$

Assume that the number of equipment failures $\{N(t), t \geqslant 0\}$ can be determined using an intensity function, $\lambda(t)$ for $t \geqslant 0$, that exhibits non-homogeneous point process, and the number of equipment failures satisfy the Conditions (i)–(iv):

(i) $N(0) = 0$.
(ii) $\{N(t), t \geqslant 0\}$ has independent increments.
(iii) $P\{N(t+\Delta t) - N(t) \geqslant 2\} = o(\Delta t)$.
(iv) $P\{N(t+\Delta t) - N(t) = 1\} = \lambda(t)\Delta t + o(\Delta t)$.

Let $h(t)$ be a continuous function for given times $0 = t_0 < t_1 < t_2 < \cdots < t_n = T$, in which the $h(t)$ denotes the loss of equipment failure within the time t. Let r denote the non-risk interest rate. Then, the expected loss in the period $[0,T]$ can be presented by

$$\text{Penalty } [0,T] \approx E\left[\sum_{i=0}^{n-1} e^{-rt_i} h(t_i) I_{[N(t)=1, t_i < t < t_{i+1}]}\right] = \sum_{i=0}^{n-1} e^{-rt_i} h(t_i) P\{N(t) = 1, t_i < t < t_{i+1}\}$$

For large n, the probability $P\{N(t) = 1, t_i \leqslant t < t_{i+1}\}$ can be approximated by

$P\{N(t) = 1, t_i \leqslant t < t_{i+1}\} \approx \lambda(t_i)(t_{i+1} - t_i), i = 0,1,2,...,n-1$, and it can be shown that

$$\text{Penalty}[0,T] = \int_0^T e^{-rt} h(t)\lambda(t)dt \quad \text{as} \quad n \to \infty. \tag{1}$$

Assume that n PMs are implemented in the period $[0,T]$. Let $\tau_n = T/(n+1)$ denote the time between two PMs. The degree of PM (or named age reduction) is denoted by $\widetilde{x} = (x_1, x_2, ..., x_n)$, where $0 \leqslant x_i < i\tau_n - \sum_{k=0}^{i-1} x_k$, $i = 1,2,...,n$. For each equipment failure, a minimal repair is adopted. Since the failure of the equipment may result in huge loss, a fast repair procedure is needed. Minimal repair is a widely used action to develop a PM policy. The minimal repair time is denoted by Y. Let $\delta > 0$ denote the repair time limit. The lessor must pay for penalty if the minimal repair time over the threshold δ, that is $Y > \delta$. Let $G(\delta) \equiv P(Y > \delta)$. Denote the failure rate function with $\lambda_n(t|\widetilde{x})$. After the i th PM, the cumulative age reduction is $x^{(i)} = \sum_{j=1}^{i} x_j$ for $i = 1,2,...,n$. The expected number of failures within $[0,t)$ can be presented by $\Lambda_n(t|\widetilde{x}) = \int_0^t \lambda_n(s|\widetilde{x})ds$. The $\lambda_n(t|\widetilde{x})$ is referred as IFR (increasing failure rate) if $\lambda_n(t|\widetilde{x})$ is increasing and is referred as DFR (decreasing failure rate) if $\lambda_n(t|\widetilde{x})$ is decreasing.

Based on the aforementioned discussions, three costs are considered in the model, namely the minimal repair cost, PM cost and penalty that the lessor must pay for overdue minimal repair time. Let $c_m(x_i)$ denote the PM cost for $1 \leqslant i \leqslant n$, c_r denote the minimal repair cost, and let $L(S(t),D(t))$ denote the penalty, where $D(t)$ are product demand within time t and $S(t)$ be the price of product at the time t. The penalties depend on the product price and demand at the time t. In this paper, we consider that $c_m(x_i)$ is an increasing function of x_i to make sense for practical applications. The expected CM cost is formulated by

$$c_r \int_0^T e^{-rt}\lambda_n(t|\widetilde{x})dt = c_r \sum_{i=1}^{n+1} \int_{(i-1)\tau_n}^{i\tau_n} e^{-rt}\lambda(t - x^{(i-1)})dt,$$

and the expected total penalty function during the period $[0,T]$ is obtained by

$$\sum_{i=1}^{n+1} \int_{(i-1)\tau_n}^{i\tau_n} e^{-rt} G(\delta) E\{L[S(t),D(t)]\}\lambda(t - x^{(i-1)})dt.$$

Because $G(\delta)$ is a constant, the loss of equipment failure within the time t can be presented by $L_\delta(S(t),D(t)) = G(\delta)L(S(t),D(t))$. The expected total cost function under the PM strategy $(n,\widetilde{x},T)$ can be obtained by

$$EC_M(n,\widetilde{x},T) = \sum_{i=1}^{n+1} \int_{(i-1)\tau_n}^{i\tau_n} e^{-rt} E\{L_\delta[S(t),D(t)]\}\lambda(t - x^{(i-1)})dt + c_r \sum_{i=1}^{n+1} \int_{(i-1)\tau_n}^{i\tau_n} e^{-rt}\lambda(t - x^{(i-1)})dt + \sum_{i=1}^{n} e^{-ri\tau_n} c_m(x_i). \tag{2}$$

The optimal PM strategy is defined as $(n^*, \widetilde{x}^*, T)$, which is a decision to minimize the expected total cost function in Eq. (2).

Let $(0,\phi)$ denote the simple PM strategy that is a maintenance strategy without implementing any PMs. A PM strategy can be established according to Theorem 1.