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A graph based group decision making approach with intuitionistic fuzzy preference relations

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ABSTRACT

Intuitionistic fuzzy preference relation (IFPR) is an efficient tool in tackling comprehensive multi-criteria group decision making (MCGDM) problems via pairwise comparisons. Based on the intuitionistic fuzzy analytic hierarchy process (IFAHP) and the best-worst method (BWM), this paper aims to put forward a novel graph-based group decision making approach called the intuitionistic fuzzy best-worst method (IF-BWM) for MCGDM. To achieve this goal, we first aggregate the individual IFPRs provided by the decision makers into a collective IFPR by the intuitionistic fuzzy weighted averaging (IFWA) operator. Afterwards, we draw the directed network according to the collective IFPR, and then design an algorithm to identify the best and worst criteria through computing the out-degrees and in-degrees of the directed network. Furthermore, to derive the weights of criteria, some mathematical models corresponding to the different definitions of consistent IFPR are developed. Finally, the procedure of the IF-BWM is proposed for practical applications and three numerical examples are given to illustrate the approach.

1. Introduction

Analytic hierarchy process (AHP), as a classic theory of measurement, was originally introduced by Saaty (1980), and has become one of the most important decision making techniques. By decomposing a complex problem into a multi-level hierarchic structure of objectives, criteria, sub-criteria and alternatives, the AHP can assist the decision maker to describe the general decision operation when it was applied to decision making. The procedure of AHP can be divided into three steps: (1) providing a fundamental scale of relative magnitudes expressed in dominance units to represent the judgments of pairwise comparisons; (2) deriving the ratio scales of relative magnitudes expressed in priority units from each set of comparisons; (3) synthesizing the ratio scales of priorities and then obtaining the ranking of alternatives (Saaty, 1990). The AHP has been applied comprehensively to solve various decision making problems, such as the U.S.-OPEC Energy Conflict (Saaty, 1979), the marketing investment (Smyth & Lecoeuvre, 2015), the evaluation of information and communication technology (ICT) business alternatives (Angelou & Economides, 2009), and so on. In the classic AHP model, the relative magnitudes of pairwise comparisons over different criteria are represented by crisp numbers within the 1-9 scale. However, in some realistic situations, people find that they encounter difficulties in assigning the crisp evaluation values to the comparison judgments due to some objective or subjective reasons such as knowledge limitation, individual interest and personal preferences, complexities and fuzziness of the things, etc. Hence, even though the AHP has been popular and simple in handling multi-criteria decision making (MCDM) problems, it is often criticized for its inability to tackle the inherent uncertainty and vagueness effectively (Xu & Liao, 2014).

In order to improve the ability of AHP, some innovative theories, such as the fuzzy set theory (Zadeh, 1965) and the intuitionistic fuzzy set (IFS) theory (Atanassov, 2012), etc. have been applied to combine with the classical AHP. Thus, a succession of extended methods under uncertain circumstances have been developed, which include the fuzzy AHP (FAHP) (Ajami & Ketabi, 2012; Chena, Hsieha, & Do, 2015; Wang, Luo, & Hua, 2008) and the intuitionistic fuzzy analytic hierarchy process (IFAHP) (Liao & Xu, 2015; Xu, 2007; Xu & Liao, 2014), etc. Concerning the FAHP, the earliest study was initiated by Van Laarhoven and Pedrycz (1983). Through directly extending the classical AHP with, respectively, triangular fuzzy numbers and trapezoidal fuzzy numbers, Van Laarhoven and Pedrycz (1983) and Buckley (1985) derived fuzzy weights and fuzzy performance scores to rank alternatives. Boender, de Graan, and Lootsma (1989) proposed a more robust

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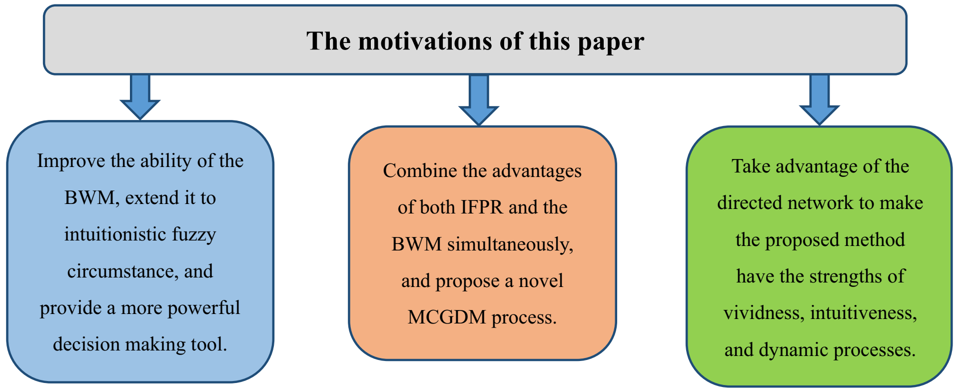

Fig. 1. The motivations of this paper.

approach to normalize the local priorities via modifying Van Laarhoven and Pedrycz's method. Later, using a row mean method, Chang (1996) developed the FAHP in the context of triangular fuzzy numbers to derive priorities for comparison ratios. After that, Wang et al. (2008) reviewed the relative AHP methods through three numerical examples, and gave two conclusions: Chang's method (Chang, 1996) is relatively easier than the other FAHP approaches and similar to the conventional AHP, and it has more comprehensive applications than other FAHP methods. Later, Kwong and Bai (2003) complemented the step about the consistency checking procedure of the pairwise comparison for FAHP (Chang, 1996), and then measured the consistency using Saaty's (1980) consistency index and consistency ratio, but it was a pity that they transformed triangular fuzzy numbers straightly into crisp numbers. As pointed out by Xu and Liao (2014), one drawback of the methods in Van Laarhoven and Pedrycz (1983) and Chang (1996) is that the transformation process possibly causes a loss of information, and hence may distort the final results. To handle both vagueness and ambiguity related uncertainties in the environmental decision making process, recently, Xu and Liao (2014) summarized the procedure of AHP to three principles: decomposition, pairwise comparison and synthesis of priorities, and extended the classic AHP and the FAHP into the IFAHP. Sadiq and Tesfamariam (2009) also applied the concept of IFS to AHP. Because the IFAHP utilizes intuitionistic fuzzy values to represent membership degrees, non-membership degrees and hesitancy degrees, it can be seen as a particular case of type 2 fuzzy set. But the triangular fuzzy numbers and the trapezoidal fuzzy numbers do not have this property and each of them only can represent one grade of membership that is crisp in the unit interval $[0,1]$ (Xu & Liao, 2014). Xu and Liao (2014)'s IFAHP proposed a new way to check the consistency which is different from the FAHP, and meanwhile, they also introduced an automatic scheme to repair the inconsistent intuitionistic fuzzy preference relation. From the difference between the AHP method and the IFAHP method, we can see that the intuitionistic fuzzy values can represent the preferences of the pairwise comparison more comprehensively, and thus the IFAHP method is more powerful in reflecting the vagueness and uncertainty. Though there are some criticisms regarding the misuse of fuzzy sets in "fuzzy AHP/ANP approaches", these improved and fashionable AHP methods can provide comprehensive and intuitional structures to combine both qualitative and quantitative criteria. They are popular in the fuzzy decision making processes, and have been applied extensively in various fields, such as environmental decision making (Sadiq & Tesfamariam, 2009), pattern recognition (Boran & Akay, 2014), teaching performance evaluation (Chena et al., 2015), etc.

Preference relations, such as fuzzy preference relations (FPRs) (Orlovsky, 1978) and intuitionistic fuzzy preference relations (IFPRs) (Xu, 2007), are considered as two of the most important forms to express pairwise comparisons in employing different types of AHP methods. The IFPR, as a powerful decision making tool, has shown advantages in handling vagueness and uncertainty due to the efficiency in expressing the imprecise cognitions of the decision makers. Furthermore, from positive and negative points of view, people can express their own opinions with IFPRs over different pairs of alternatives. An IFPR gives the degrees of both membership and non-membership that an alternative is prior to another. Xu (2008) proposed the intuitionistic fuzzy weighted averaging (IFWA) operator, and later, Liao and Xu (2014) discussed the consistency of the fused IFPR and developed the group IFAHP. Nevertheless, in terms of visual intuition and easy manipulation, the existing decision making procedures have not enough abilities for considering both two qualities above.

The directed network is an effective tool reflecting the relationship between objects. It consists of some nodes and directed edges connecting pairs of vertices. Usually, the approaches based on the graph theory (Bondy & Murty, 1976) have the advantages of vividness, intuitiveness, and dynamic processes over other methods of dealing with the MCGDM problems. Recently, Rezaei (2015) proposed a novel best-worst method (BWM) for the MCDM problems, which can be taken as an enhancement of the traditional AHP and FAHP methods. With the BWM, the decision maker does not need to conduct pairwise comparisons between all criteria but only needs to identify the most desirable criterion as the best one and the least desirable criterion as the worst one, and then makes pairwise comparisons between the best/worst criterion and the other criteria. Then the decision maker constructs a max-min mathematical model to determine the weights of different criteria, and gives a new definition of consistency ratio to check the reliability of the method. However, it is not easy for us to determine which criterion is the best or worst one when the number of criteria is very large, and their approach is improper under uncertain circumstances. In this paper, we combine the advantages of two decision making tools: the directed network and the BWM method. On the one hand, we take advantage of the directed network to help us rank the criteria in the MCGDM problems; on the other hand, we try to extend the BWM to accommodate intuitionistic fuzzy circumstances. The above two aspects can make the process of decision making more vivid, intuitive, and dynamic than other decision making methods.

No matter what kind of job we have, and no matter where we live, we always meet a variety of evaluation and selection problems in our personal life and occupation career. In the case of the