Computers & Industrial Engineering 110 (2017) 322–332

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

The robust uncapacitated multiple allocation p-hub median problem

El-Ghazali Talbi, Raca Todosijević *

INRIA Lille-Nord Europe, 40 Avenue Halley, 59650 Villeneuve d'Ascq, France

ARTICLE INFO

Article history:
Received 29 September 2016
Received in revised form 8 June 2017
Accepted 10 June 2017
Available online 11 June 2017

Keywords:
Uncertainty
Robustness measure
p-hub
Hub median
Multiple allocation

ABSTRACT

In this paper we study the uncapacitated multiple allocation p-hub median problem and propose several ways to deal with the uncertainty that may occur in the flow. More precisely, the paper introduces a new way to quantify the robustness of a solution in the presence of uncertainties. The main characteristic of the newly proposed robustness measure is its ability to provide a solution which is robust for any realization of the number of changes that may occur. An extensive empirical study is conducted to justify the need for the new robustness measure as well as the potential gains it brings.

1. Introduction

Hubs are a special kind of facilities used in real-world transportation networks to collect, consolidate, transfer and distribute flows. In hub networks, transportation between non-hub nodes is accomplished indirectly, via the hub nodes to which non-hub nodes are assigned. This feature represents the main advantage of hub networks; namely, cost required for their establishment is lower than the cost required for a network where any two nodes are directly connected. Early applications of hub location problems involved the design and management of telecommunication and transportation systems, such as computer and satellite networks, logistical systems and airline networks. Nowadays, the concept of hub networks is used in many new areas, such as freight transportation, fast delivery systems, public transit and maritime industry. An overview of classification, solution approaches, and applications of hub location problems can be found in Farahani, Hekmatfar, Arabani, and Nikbakhsh (2013).

A hub location problem consists of selecting the hub locations and allocating non-hub nodes to the established hubs. In general, most of the hub location problems are NP-hard. In some situations, the allocation part of the problem (i.e., assigning non-hub nodes to the hubs) remains NP-hard even if the locations of the hubs are known (Alumur & Kara, 2008).

Various criteria can be used to classify hub location problems. One of them is based on the allocation scheme, that defines the way of allocating non-hub nodes to hubs. Two main allocation schemes are distinguished: single allocation (each non-hub node is allocated to exactly one established hub) and multiple allocation (each non-hub node may be allocated to multiple hubs). Recently, a r–allocation scheme has been introduced in Yaman (2011), which has allowed to assign each non-hub node to at most r hubs. Another criterion is based on the objective function. The most common hub location problems are those that consider center or median objectives. The hub median problem consists of locating hub nodes and assigning non-hub nodes to hubs so that the total transportation cost between all origin-destination (O-D) pairs is minimized (see e.g., Ernst & Krishnamoorthy, 1996; Ilić, Urošević, Brimberg, & Mladenović, 2010; Kratica, 2013; Martí, Corberán, & Peiró, 2015; Peiró, Corberán, & Martí, 2014; Todosijević, Urošević, Mladenović, & Hanafi, 2015). On the other hand, the hub center problem seeks to minimize the maximal distance between origin-destination pairs (see e.g., Brimberg, Mladenović, Todosijević, & Urošević, 2015a, 2015b; Ernst, Hamacher, Jiang, Krishnamoorthy, & Woeginger, 2009; Gavriliouk, 2009; Meyer, Ernst, & Krishnamoorthy, 2009).

In this paper, we focus on the uncapacitated multiple allocation p-hub median problem (UMApHMP). It consists in choosing p hub locations from a set of nodes with pairwise traffic demands in order to route the traffic between the origin-destination pairs such that the total cost between origin-destination pairs is minimum. The UMApHMP was introduced in O'Kelly and Morton (1987) and formulated as a quadratic integer program with a non-convex objective function. Later, several mixed integer formulations have been proposed such as those in Campbell (1994, 1998, 2012). Regarding solution approaches for the UMApHMP, both

* Corresponding author.
E-mail addresses: el-ghazali.talbi@univ-lille1.fr (E.-G. Talbi), racatodosijevic@gmail.com (R. Todosijević).

http://dx.doi.org/10.1016/j.cie.2017.06.017
0360-8352/

exact and heuristic approaches have been proposed. Exact approaches are based on branch and bound framework (see e.g., Aykin, 1995; Boland, Krishnamoorthy, Ernst, & Ebery, 2004; Ernst & Krishnamoorthy, 1998). However, large scale instances remain still elusive for exact approaches and therefore heuristic algorithms ranging from simple heuristics to those more advanced (hybrid) have been proposed (see e.g., Aykin, 1995; Ernst & Krishnamoorthy, 1998; Kratica, 2013; Liu, Wu, & Luo, 2011; Liu, Wu, & Luo, 2012; Milanović, 2010; Stanimirović, 2008).

All of the above studies on UMApHMP deal with the deterministic formulation of the problem. However, in practice, flows to be transferred are random and thus an optimal solution for a certain realization is not necessarily optimal for other realizations. For example, in postal and cargo services, the demand is uncertain and cannot be estimated accurately. Therefore, the purpose of this paper is to provide decision makers with one possible way to deal with the uncertainty that appears in the hub networks and to propose an efficient way to build a solution, which is robust for any realization of the flow. In sum, the main contributions of the paper are threefold:

- In order to deal with flow uncertainty, a new robustness measure that includes the robustness measure from Bertsimas and Sim (2003) as a special case is proposed. The main advantage of the newly proposed robustness measure is to provide a solution which is robust for any realization of the number of changes. The robustness measure from Bertsimas and Sim (2003) provides a robust solution for a predefined number of changes, Γ, that may occur. However, estimating the value of Γ accurately might be hard in practice. For example, suppose that we implement a robust solution defined with respect to a certain (most likely) value of Γ. In practice, some other value of Γ may be realized making the implemented solution to be of poor quality. In order to overcome this situation, we propose a new robustness measure where all possible realizations of Γ values are taken into account.
- The UMApHMP under uncertainty is studied and corresponding exact and heuristic solution approaches are developed.
- Numerical experiments, using the benchmark p-hub instances, are performed in order to demonstrate the need for the newly proposed robustness measure. The numerical results reveal how uncertainty and different robustness measures may affect the problem solution.

The remaining of the paper is organized as follows. In the next section we describe the deterministic UMApHMP. In Section 3, we describe a new robustness measure and develop the robust counterpart of UMApHMP. In Section 4, we propose a solution approach to tackle this robust model. Section 5 is devoted to an empirical analysis conducted on the benchmark instances for p-hub location problems in order to assess benefits of using newly proposed robustness measure. Finally, Section 6 concludes the paper and indicates some possible research directions.

2. Deterministic multiple allocation *p*-hub median problem

The UMApHMP is defined on a complete symmetric graph $G = (N, E)$, where $N = \{1, 2, \ldots, n\}$ represents a set of nodes and $E = \{(i,j) : i,j \in N\}$ denotes a set of arcs. No capacity restrictions on arcs $(i,j) \in E$ are imposed. The transportation cost per unit of flow on arc $(i,j) \in E$ is denoted as c_{ij}. The demand t_{ij} that has to be transferred from node i to node j is given. UMApHMP aims at selecting exactly p nodes from the set N to be hubs, where p is a predetermined parameter, so that the total transportation cost is minimized, assuming that any non-hub node may use any hub node to communicate with other nodes and the flows can be sent and received through more than one hub (multiple allocation scheme). The transportation cost from node $i \in N$, assigned to hub k, to node $j \in N$, assigned to hub l is calculated as:

$$d_{ijkl} = \gamma c_{ik} + \alpha c_{kl} + \delta c_{lj},$$

where parameters γ, α and δ are unit rates for collection (origin-hub), transfer (hub-hub), and distribution (hub-destination), respectively. In general, parameter α is used as a discount factor to provide reduced unit costs on arcs between hubs, so $\alpha < \gamma$ and $\alpha < \delta$. Such parameter configuration is required to accurately model the real world situation where collection, distribution, and transfer methods are all different. In the postal delivery networks, for example, the different modes for collecting, transferring, and distributing mails are used (see Ernst & Krishnamoorthy, 1996).

Mathematically, the problem can be formulated as follows. Let variable z_k be equal to 1 if the node k is a hub, and 0 otherwise. Furthermore, let variable f_{ijkl} be the proportion of the traffic t_{ij} from node i to node j that travels along the path $i - k - l - j$, where k and l denote hubs. Then, the uncapacitated multiple allocation p-hub median problem may be stated as follows (O'Kelly, Bryan, Skorin-Kapov, & Skorin-Kapov, 1996; Skorin-Kapov, Skorin-Kapov, & O'Kelly, 1996):

$$\min_{f_{ijkl}} \sum_{i \in N} \sum_{j \in N} \sum_{k \in N} \sum_{l \in N} t_{ij} d_{ijkl} f_{ijkl} \tag{1}$$

subject to

$$\sum_{k \in N} z_k = p \tag{2}$$

$$\sum_{k \in N} \sum_{l \in N} f_{ijkl} = 1 \quad \forall i,j \in N \tag{3}$$

$$\sum_{l \in N} f_{ijkl} \leqslant z_k \quad \forall i,j,k \in N \tag{4}$$

$$\sum_{k \in N} f_{ijkl} \leqslant z_l \quad \forall i,j,l \in N \tag{5}$$

$$f_{ijkl} \geqslant 0 \quad \forall i,j,k,l \in N \tag{6}$$

$$z_k \in \{0,1\} \quad \forall k \in N \tag{7}$$

Constraint (2) ensures that exactly p hubs are selected. Constraints (3) impose that the traffic between each pair of nodes i,j is entirely routed, while constraints (4) and (5) ensure that the routing is accomplished only through opened hub nodes. Note that since all the costs are linear and there is no restrictions on capacity, the flow in an optimal solution will not split as one would always route through the cheapest option.

3. The robust multiple allocation *p*-hub median problem

Stochastic programming, introduced in the mid 1950s by Dantzig (1955), represents an approach where the data are modeled by means of random variables from known probability distributions. In this case, the optimization problem is stochastic and the objective function interprets the probabilistic structure of the model. Usually, the objective function may quantify an expected cost, the probability of violation of some constraints, variance measures, and so on. However, the two main drawbacks of such an approach are: (i) determining the exact distribution of the data, and thus enumerating scenarios that capture this distribution is rarely satisfied in practice, and (ii) the size of the resulting optimization model increases drastically, with the number of scenarios, which poses substantial computational challenges. For applications of stochastic programming we refer the reader to Bianchi, Dorigo, Gambardella, and Gutjahr (2009), while recent applications to hub location problems may be found in e.g., Contreras, Cordeau, and Laporte (2011), Sadeghi, Jolai, Tavakkoli-